Paying for Beta: Embedded Leverage and Asset Management Fees

Steffen Hitzemann, Stanislav Sokolinski and Mingzhu Tai*

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Abstract

This article studies the effects of leverage constraints on asset management fees. We present a new model in which constrained investors delegate capital to asset managers. In the equilibrium, risk-seeking investors choose high beta managers who charge high fees for providing embedded leverage. Our empirical results in the sample of the U.S. equity mutual funds are consistent with the model's three novel predictions: 1) fees increase in market beta, but only when beta is larger than one, 2) this relation is stronger when leverage constraints tighten, and 3) fund net alpha declines in beta faster than its gross alpha. Our study suggests that willingness to pay for embedded leverage helps explain net-of-fees underperformance of actively managed funds.

Keywords: Leverage; Financial Intermediation; Mutual Funds

*Steffen Hitzemann (hitzemann@business.rutgers.edu) and Stanislav Sokolinski (ssokolinski@business.rutgers.edu) are at Rutgers Business School, Mingzhu Tai (taimzh@hku.hk) is at The University of Hong Kong.

1 Introduction

Many investors delegate portfolio construction to professional money managers and pay fees for asset management service. The extent of delegation and the resultant fee revenue have grown significantly over the last three decades. French (2008) reports that individual investor holdings of U.S. common equity declined from 47.9% in 1980 to only 21.5% in 2007 while open-end mutual fund holdings increased from 4.6% to 32.4% over the same period. At the same time, investors sacrificed about 10% of their annual real return for asset management fees and transaction costs.[1] Fee determination represents a long-standing puzzle for financial economists since many funds charge fees which are significantly higher than their risk-adjusted returns.[2] In this article we present and test a novel explanation for this puzzle: asset managers can charge fees for the provision of leverage to investors who face borrowing constraints, even if asset managers lack the ability to beat the market.

To understand how borrowing constraints might affect asset management fees, consider the following example. Imagine two investors who do not assemble portfolios on their own but can easily obtain leverage. The risk-seeking investor seeks to obtain a portfolio with the market beta of 1.5 and the risk-averse investor wants to achieve a beta of 0.5. According to the capital asset pricing model, the first investor borrows 50% of her wealth and invests 150% of it in the market index fund while the second investor equally splits her holdings between the market index fund and the risk-free asset. But if the risk-seeking investor cannot borrow, she has to find an asset manager who can assemble a portfolio with beta of 1.5 on her behalf. Such a manager can charge a markup for providing beta greater than one even at the absence of any ability to beat the market. Importantly, the risk-averse investor does not face borrowing constraints as she still can achieve her desired beta of 0.5 solely relying on the market index fund. As a result, low-beta asset managers cannot charge markups similar to high-beta managers.

To sharpen this intuition, we present a new model in which constrained investors delegate capital to asset managers. In the model, investors differ in their risk aversion and they do not construct portfolios on their own. Asset managers differ in the amount of leverage that they provide as measured by market beta. Critically, asset managers do not possess any

[1] In 2018, only the U.S equity mutual fund investors paid more than $50B in fees. This calculation is based on the Investment Company Institute 2019 report. The total mutual fund industry assets under management as of December 2018 equal to $17.7T where equity funds represent 52% of assets. The value-weighted expense ratio for the equity funds equals to 0.55%.

[2] For early evidence on underperformance of actively managed funds, see Jensen (1968), Ippolito (1989) and Gruber (1996). For the recent advancements, see, for example, Fama and French (2010), Del Guercio and Reuter (2014) and Berk and van Binsbergen (2015).

ability to beat the market: all managers invest in the market portfolio with different degrees of leverage. Managers compete on fees and each investor needs to choose her preferred asset manager. If investors are not willing to delegate to asset managers at the given fee structure, they can always decide to invest in the market index fund for a trivial fee.

In the equilibrium, the risk-seeking investors invest with high-beta asset managers. The demand for leverage grants managers with betas greater than one local monopoly power and allows to charge high fees. At this range of betas, fees progressively increase with beta such that the most risk-seeking investors pay very high fees due to their high demand for leverage. At the same time, the risk averse investors seek betas smaller than one and do not face leverage constraints. Instead, they split their portfolios between the cheap market index fund and the risk-free asset. As a result, asset managers with betas smaller than one do not possess any leverage-based monopoly power and cannot charge fees higher than the market index fund.

The model immediately delivers a key new testable hypothesis: in the cross-section of funds, asset management fees increase in fund beta when beta is larger than one, but they are flat in beta when beta is smaller than one. Furthermore, the model predicts that the relationship between beta and fees, at the range of betas greater than one, becomes stronger when leverage constraints tighten. Finally, our model has an implication for the net-of-fee performance of actively managed funds. Since we suggest that investors pay fees not only for performance but also for embedded leverage, we expect funds to underperfrom more due to high fees when leverage provision becomes especially valuable. In particular, the model predicts that net alpha declines with fund beta faster than gross alpha, when beta is larger than one. This result helps explain why certain funds charge fees substantially higher than their gross alphas, contributing to underperformance of the average actively managed fund.

We go on to test the model's implications in the sample of U.S. domestic equity mutual funds. We first examine the cross-sectional relationship between fund fees and fund beta for different ranges of betas as guided by the model. Implementing a variety of cross-sectional tests and controlling for the known determinants of fees, we confirm our first hypothesis: when beta is larger than one, fund fees increase with fund beta. At the same time, when fund beta is below one, the relation between beta and fees becomes economically and statistically insignificant. The relation between betas and fees for betas above one is economically meaningful: when fund beta increases from 1 to 1.7, the 99th percentile of beta distribution, fund fees increase by 25 basis points. This relationship also stands as economically significant relative to the effects of other determinants of fund fees. For example, a one-standard deviation increase in fund beta is associated with an increase in fees of 8 basis points. At the same time, an increase of one standard deviation in log fund size is associated with a reduction of 11 basis points in fees, while an increase of one standard deviation in log fund age is associated with an increase of 3 basis points in fees.

We next test whether the relationship between beta and fees, for funds with betas greater than one, becomes stronger when leverage constraints tighten. This prediction provides additional support to our model suggesting that the relationship is specifically driven by demand for leverage. We present two series of tests. We first study the differences between institutional investors and retail investors. Since retail investors may face tighter leverage constraints, we expect the relationship between beta and fees to be stronger for this group of investors.[3] We examine the differences between institutional and retail share classes and confirm the model's second prediction. When a share class is offered to retail investors, the fund family charges almost twice higher fees for the same beta relative to what it charges institutional investors.

We present our second series of tests studying the cross-section of funds that start their operations in different time periods. In particular, we compare funds that launched in periods of tight leverage constraints with the funds launched in less constrained periods. In these tests, we employ time variation in measures of the tightness of leverage constraints used in the literature. Specifically, we use variation in betting-against-beta (BAB) factor from Frazzini and Pedersen (2014), intermediary capital risk (ICR) factor from He, Kelly, and Manela (2017), as well as leverage constraint tightness (LCT) measure from Boguth and Simutin (2018). We expect the relation between beta and fees to be stronger in the cross-section of funds launched in periods of tight leverage constraints relative to funds launched in less constrained periods. Consistent with this hypothesis, we find that funds launched during constrained periods exhibit a stronger relationship between fund beta and fund fee. Our tests confirm the model predictions and produce economically significant results. Depending on the measure of tightness, funds introduced in constrained periods charge two to four times more per unit of beta relative to funds introduced in less constrained periods.

Finally, we empirically examine the implications of our model for fund net-of-fees performance. Using portfolio sorting, we first document that the increase in market beta is associated with lower gross alphas. The difference in gross alphas between low and high beta fund portfolios equals to 37 basis points, however this difference is not statistically significant at the 10% level. This result is consistent with the analysis of individual stocks in Frazzini and Pedersen (2014) who show that high-beta stocks have lower alphas.

[3] Frazzini and Pedersen (2014) show that individual investors are more likely to hold high beta stocks, consistent with the intuition that they face tighter leverage constraints.

At the same time, our model and previous empirical results imply that net alphas should decline even further. Since we have shown that fund fees increase in beta for betas above one, we expect net alphas to decline with beta even faster relative to gross alphas. Consistently with this prediction, the difference in net alphas between low and high beta fund portfolios amounts to 60 basis points and it is statistically significant at the 5% level. 40% of this difference (23 basis points) is purely driven by the difference in fees between the funds. At the same time, we show that high beta funds generate higher net returns relatively to low beta beta funds. The combined evidence implies that investors pay high fees to obtain access to high returns of high beta funds while facing lower net alphas. Our results suggest that demand for leverage helps explain low net-of-fees performance of actively managed funds as captured by fund net alpha.

1.1 Literature

This article contributes to the literature that studies how investors delegate capital and how management fees are determined. In addition to early work by Berk and Green (2004), recent research includes Gil-Bazo and Ruiz-Verdu (2008), Cuoco and Kaniel (2011), Guerrieri and Kondor (2012), Kaniel and Kondor (2012), Glode (2011), and Gârleanu and Pedersen (2018). These theoretical papers focus on the interplay between fund performance and fees by studying how investors reward asset managers. Our results complement this work by emphasizing the importance of embedded leverage that asset managers can provide even if they lack any ability to beat the market. We also contribute to the empirical literature that examines determinants of mutual fund fees such as Christoffersen and Musto (2002), Gil-Bazo and Ruiz-Verdú (2009), and Khorana et al. (2008).

A recent strand of literature argues that managers can charge fees for providing access to financial markets even at the absence of superior performance (Gennaioli, Shleifer, and Vishny 2014, Gennaioli, Shleifer, and Vishny 2015). Our paper follows their general idea of delegation, but takes a different perspective. In their model, asset managers also compete on fees but all managers charge the same fee in equilibrium. The ability to charge fees arises from different levels of trust that investors place in different money managers. In contrast, in our model equilibrium fees vary across managers within a given asset class and there is no differentiation on trust. Our theory also distinctively predicts that the positive relationship between beta and fees can arise only for high-beta asset managers within a given asset class.

Our article also speaks to a small nascent literature that studies the effects of embedded leverage. Frazzini and Pedersen (2012) and Frazzini and Pedersen (2014) show that leverage constraints affect asset prices and Boguth and Simutin (2018) argue that the variation in mutual fund market beta captures the demand for leverage. In a recent work, Lu and Qin (2019) use leveraged funds to estimate shadow costs of leverage while Dam, Davies, and Moon (2019) show that the demand for leverage contributes to discounts on closed-end funds. Our work extends this literature by focusing on the effect of the demand for leverage on fees of financial products and the resultant net performance.

We organize the rest of the paper as follows. In Section 2, we present and solve our model generating the key testable hypotheses. In Section 3, we describe our data and methodology. Section 4 takes the model to the data and examines the testable hypotheses, and Section 5 concludes.

2 Model

2.1 Setup

Our model has two dates and two types of agents: asset managers and investors. At time 0, asset managers set fees and investors choose managers since we assume that investors do not invest on their own.[4] At time 1, managers liquidate their funds and distribute net-of-fees assets to the investors. There is a number J of asset managers who manage funds with different market betas $0 < \beta_0 < \beta_1 = \beta_M < \ldots < \beta_J$, where β_M stands for the asset manager who offers a market index fund. Asset managers charge fees ϕ_j per dollar invested. The risk-free asset delivers a return of R_f and the returns of the asset managers follow the CAPM: a fund with β_j has an expected (before-fee) excess return of $\beta_j \mu_M$, with $\mu_M = E[R_M]$, and volatility $\beta_j \sigma_M$ with $\sigma_M^2 = Var[R_M]$.

There is a unit measure of investors. Investors have CARA preferences with risk aversion γ_i and each investor is endowed with a unit of wealth. Investors decide to invest a fraction ω_i of their wealth with one asset manager of their choice, while the remaining wealth is invested into the risk-free asset. Investors face borrowing constraints, $\omega_i \leq l$: in particular, a fraction ψ of the investors is strictly borrowing constrained, $l = \underline{l} = 1$, while a fraction $1 - \psi$ has to fulfill a relaxed constraint with $l = \bar{l} > 1$.

There is perfect supply-side competition for market index funds with $\beta_M = 1$. As a result, the fee ϕ_M on the market index fund equals marginal production/management costs, which we assume to be zero.[5] All asset managers with betas different from one are subject

[4] This assumption is typical for theories of delegated asset management. See, for example, Cuoco and Kaniel (2011), Gennaioli, Shleifer, and Vishny (2014), and Gennaioli, Shleifer, and Vishny (2015).

[5] Our results are not sensitive to this simplifying assumption. We discuss this point when we present the main results.

to monopolistic competition. Intuitively, all market index funds are similar and in perfect competition with each other, while other asset managers differ in their betas and therefore have monopoly power to some extent over their respective clienteles.

Investors' Problem Each investor decides how much to invest into risky assets and also chooses an asset manager. Formally, investor i solves the problem

$$\max_{\beta_j, \omega_i} \; \omega_i(\beta_j \mu_M - \phi_j) + R_f - \frac{\gamma_i}{2} \omega_i^2 \beta_j^2 \sigma_M^2 \tag{1}$$

with $\beta_j \in \{\beta_0, \beta_1, \ldots, \beta_J\}$ and $\omega_i \in [0, l]$, subject to the given borrowing constraint.

Asset Managers' Problem Each asset manager maximizes revenues that she generates from fees. Asset manager j solves the problem

$$\max_{\phi_j} \; \phi_j AUM_j(\phi_j), \tag{2}$$

where AUM_j are the assets under management that are allocated to j when the fee is set to ϕ_j. As asset managers operate under monopolistic competition, they take the investors' demand function as given when maximizing revenues.

2.2 Investor Choice and Fund Assets Under Management

We characterize the investors' investment choices, given the universe of available managers with different betas and the assumption that each investor picks exactly one manager to invest in. On the manager side, we do not explicitly model entry and exit, but we assume that an asset manager j only survives in equilibrium if a non-zero mass of investors prefers j over all other managers in the universe.

Investor Choice Without loss of generality, assume that investor i decides to invest with asset manager j. Then the first order condition for the weight of the risky investment is

$$\widetilde{\omega}_i^j = \frac{\beta_j \mu_M - \phi_j}{\beta_j^2 \gamma_i \sigma_M^2}, \tag{3}$$

and i chooses her investment as $\omega_i^{j^*} = \min\{\widetilde{\omega}_i^j, l\}$ due to the borrowing constraint.

We start characterizing the investor's decision by showing that investors do not invest with asset managers whose fees are too high, either in an absolute sense or relative to other managers. All the proofs are provided in Appendix A.

Proposition 1. *[Dominated Funds] Investors do not invest into funds j with*

1. $\phi_j \geq \beta_j \mu_M$ *or with*

2. $\phi_j > \phi_k$ *for a fund with* $\beta_j < \beta_k$.

The Proposition provides a necessary condition for asset managers to have positive assets under management and to survive in equilibrium. The first part shows that no investor is willing to invest with a manager whose expected after-fee excess return $\beta_j \mu_M - \phi_j$ is smaller or equal zero. The second part of the Proposition lays out the basic logic for our main result because it shows that equilibrium fees must be non-decreasing in betas. Intuitively, the fees of asset managers with smaller betas are bounded by the fees of higher-beta managers. It happens because investors can always synthesize a lower-beta fund by investing in a fund with higher beta and holding a cash position. This argument does not apply the other way round: investors cannot synthesize a high beta fund by a leveraged investment in a lower-beta fund due to the borrowing constraints. As a result, asset managers with low betas cannot charge higher fees than asset managers with high betas.

We proceed to characterize the investment decision of an individual investor in detail, given investor risk aversion γ_i. By comparing the utility provided by two given funds j and k, with $\beta_j > \beta_k$ and optimal investment weights ω_i^{j*} and ω_i^{k*}, we show that investors prefer fund j over k if their risk aversion is below a certain threshold, which we denote by $\overline{\gamma_{jk}}$. For ease of exposition, we present our result for the case that the condition

$$\beta_j \left((\beta_k - \beta_j)^2 \mu_M + 2\beta_j \phi_k \right) < \left(\beta_k^2 + \beta_j^2 \right) \phi_j \tag{4}$$

holds, for which the "cut-off" $\overline{\gamma_{jk}}$ is linear in betas and fees. Appendix A discusses the general case.

Proposition 2. *[Risk Aversion and Fund Preference] Under the given condition, investor i with borrowing bound l prefers fund j over fund k, with $\beta_j > \beta_k$, if and only if $\gamma_i < \overline{\gamma_{jk}}$, with*

$$\overline{\gamma_{jk}} = 2 \frac{\mu_M(\beta_j - \beta_k) - (\phi_j - \phi_k)}{(\beta_j^2 - \beta_k^2)\sigma_M^2 l}. \tag{5}$$

Figure 1 illustrates this result by showing combinations of an investor's risk aversion γ_i and the fee ϕ_j for which an asset manager j dominates the market index fund with $\beta_M = 1$ and $\phi_M = 0$. The right plot shows the case of investor who face relatively weak borrowing constraints ($l = 2$). The yellow area stands for the region in which an asset manager with $\beta = 1.3$ is preferred to the market index fund. Investors with very low risk aversion prefer higher-beta asset managers over the market index funds even if asset managers charge very high fee. It happens because less risk-averse investors have high demand for leverage. The fee at which a higher-beta fund j is preferred, declines as investor risk aversion increases.

We can also compare the effects of investor risk aversion on fees across asset managers with different beta. The blue area represents the region in which an asset manager with lower beta ($\beta = 1.1$) is preferred to the market index fund. We observe that the yellow area overlays the blue area. This observation shows that managers with higher beta can afford to set higher fees and can still be strictly preferred by some investors. In particular, as risk aversion declines, an investor is willing to pay for high beta asset managers significantly more even if lower beta managers are available.

Finally, we can compare the role of tightness of borrowing constraints. The left plot reproduces the effects for investor who face strict constraints ($l = 1$). In the strict case, investors cannot attain leverage by any means. As a result, even investors with moderate risk aversion prefer higher-beta asset managers to the market index fund if the fee is not too high.

At this point, we have obtained a detailed characterization of the γ-ϕ-constellations for which an individual investor prefers an asset manager j over asset manager k, showing that low risk aversion investors prefer higher-beta funds. We can extend this logic and further show that in equilibrium investors sort into funds by their betas, and the corresponding investor clienteles are formed based on their risk aversion:

Proposition 3. *[Investor Clienteles] For all funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$, it holds $\overline{\gamma_{j_2k}} < \overline{\gamma_{j_1k}}$ and $\overline{\gamma_{j_2j_1}} < \overline{\gamma_{j_1k}}$ in equilibrium, i.e., funds with higher betas are chosen by investors with lower risk aversion.*

Figure 2 illustrates this result. We observe that in equilibrium asset managers with different betas offer their services to different types of investors. In particular, investors with the lowest risk aversion choose the asset manager with the highest beta, up to a certain cut-off point, after which the second-least risk-averse clientele chooses the fund with the second-highest beta, and so on.

Finally, Propositions 2 and 3 allow us to compute the assets under management of fund j, dependent on the fee ϕ_j. In particular, it is given by

$$AUM_j(\phi_j) = \int_{\overline{\gamma}_{j+1,j}}^{\overline{\gamma}_{j,j-1}} f(\gamma_i) d\gamma_i, \tag{6}$$

where the integration bounds are defined in line with Proposition 2 and $f(.)$ is the probability density for the risk aversion in the investor population. Accordingly, we utilize the fact that asset manager j attracts investors whose risk aversion is below the threshold $\overline{\gamma}_{j,j-1}$ at which the manager with the next-lower beta is dominated, but larger than the value $\overline{\gamma}_{j+1,j}$ at which manager j is dominated by the manager with the next-higher beta.

2.3 Equilibrium

An equilibrium is a combination of fees $\phi_0, \phi_1, \dots, \phi_J$ for the asset managers such that, for optimal investor choices, the optimality condition (2) is fulfilled for all asset managers. To solve for the model equilibrium explicitly, we need to make an assumption on the probability distribution of γ_i, and we assume that γ_i is equally distributed on $[\underline{\Gamma}, \overline{\Gamma}]$. The model can then be solved analytically for the case where condition (4) is fulfilled. For other cases, we can efficiently compute the equilibrium numerically. In the analytical case, the first order conditions obtained from the fund manager optimization problems (2) constitute a linear equation system $A\phi = b$, where ϕ is the vector of all fund fees and A is a tridiagonal matrix.

Let us explicitly demonstrate and explore the equilibrium solution for the case of four funds with betas $0 < \beta_0 < \beta_1 = \beta_M = 1 < \beta_2 < \beta_3$.[6] As the first step, recall that according to our assumption, there is perfect supply-side competition for the market index funds with $\beta_1 = \beta_M = 1$, such that the corresponding fee is equal to the marginal management cost which is zero. As a result, we set $\phi_M = 0$.[7] From Proposition 1, it directly follows that the fee ϕ_0 for the asset manager with $\beta_0 < 1$ is zero too; otherwise, it would always be optimal for investors to invest in the market index fund and cash instead, to synthesize the fund with β_0 at zero fees. The same argument holds for potential additional asset managers with beta smaller than one, such that overall we obtain that fees are flat in betas for $\beta < 1$.

It therefore remains to solve for the fees ϕ_3 and ϕ_2 of the funds with $\beta_3 > \beta_2 > 1$, which are set by their managers under monopolistic competition. The revenue maximization

[6] The linear equation system for an arbitrary number of funds J is provided in Appendix A.

[7] We assume for simplicity that the marginal production/management cost of the market index fund is strictly zero, such that we obtain $\phi_M = 0$ for this highly competitive segment. The actual fee charged by the cheapest market index funds is slightly higher than this, such as 3 basis points in the case of Vanguard. Assume that the market index fund has a non-zero fee, for example, 3 basis points. Condition 2 of Proposition 1 implies in this case that each fund j with $\beta_j < 1$ has to fulfill $\phi_j \leq 0.0003\beta_j$ in order not to be dominated by the market index fund. Therefore, the fees for this range of betas are not strictly bounded to zero anymore, but are still bounded by the market index fund's fee. In this case, fees can therefore theoretically be 0 basis points for some fund with beta smaller than one, and increase to the level of 3 basis points for $\beta = 1$. Nevertheless, our calibrated results show that the model predicts a much stronger increase in fees for beta greater than one for realistic numerical examples, such that the increase of 3 basis points for betas smaller than one is still relatively flat.

problems (2), in which we insert the assets under management computed according to (6) with uniformly distributed γ_i, are obtained as

$$\begin{aligned} &\max_{\phi_2}\ \phi_2 \cdot \frac{1}{\overline{\Gamma} - \underline{\Gamma}} \left(2\frac{\mu_M(\beta_2 - \beta_M) - (\phi_2 - \phi_M)}{(\beta_2^2 - \beta_M^2)\sigma_M^2} - 2\frac{\mu_M(\beta_3 - \beta_2) - (\phi_3 - \phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} \right), \\ &\max_{\phi_3}\ \phi_3 \cdot \frac{1}{\overline{\Gamma} - \underline{\Gamma}} \left(2\frac{\mu_M(\beta_3 - \beta_2) - (\phi_3 - \phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} - \underline{\Gamma} \right), \end{aligned} \tag{7}$$

assuming for now that all investors face strict borrowing constraints (i.e., $\psi = 1$, or equivalently, $\bar{l} = 1$) for ease of exposition. All the investors with low enough risk aversion, down to the lowest risk aversion $\underline{\Gamma}$, prefer the β_3 manager over the β_2 manager. These investors invest in the β_3 fund since there is no higher beta fund available. Another group of investors invests with the β_2 manager. These investors have higher risk aversion relative to the first group and prefer to invest with the β_2 manager as opposed to the β_3 fund. At the same time, these investors still have low enough risk aversion such that they do not prefer the market index fund. The rest of the investors choose the market index fund. Fund managers maximize revenues as given by the mass of these respective clienteles, which depend on the fund fees they set, times the respective fees.

Taking the derivatives of the fund managers' objective functions by ϕ_2 and ϕ_3, respectively, and setting them to zero, yields the corresponding first order conditions

$$\begin{aligned} 2\frac{\mu_M(\beta_2 - \beta_M) - (2\phi_2 - \phi_M)}{(\beta_2^2 - \beta_M^2)\sigma_M^2} - 2\frac{\mu_M(\beta_3 - \beta_2) - (\phi_3 - 2\phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} = 0, \\ 2\frac{\mu_M(\beta_3 - \beta_2) - (\phi_3 - 2\phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} - \underline{\Gamma} = 0. \end{aligned} \tag{8}$$

Recall that $\phi_M = 0$, and we can solve the given system of two equations for the two fee variables, ϕ_2 and ϕ_3. The solution can be written as

$$\begin{aligned} \phi_2 - \phi_M &= \frac{1}{C}(A_1\mu_M - \frac{1}{2}B_1\underline{\Gamma}\sigma_M^2), \\ \phi_3 - \phi_2 &= \frac{1}{C}(A_2\mu_M - \frac{1}{2}B_2\underline{\Gamma}\sigma_M^2), \end{aligned} \tag{9}$$

where the constants $A_1, A_2, B_1, B_2, C > 0$ follow from the vector of betas and are defined in Appendix A.

We are now ready to explore the model equilibrium and to derive the relation between beta and fees which is driven by demand for leverage. In (9), for both expressions the μ_M term is greater than the negative σ_M^2 term for all reasonable combinations of the given parameters.[8] It follows that for beta greater than one, investors pay for beta. We summarize this result and its implications in the following proposition.

Proposition 4. *[Paying for Beta] Suppose* $0 < \beta_0 < \beta_1 = \beta_M = 1 < \beta_2 < \beta_3$. *In this case:*

(i) $\phi_2 - \phi_M > 0$ *and* $\phi_3 - \phi_2 > 0$. *An increase in manager beta leads to an increase in fees if beta is larger than one.*

(ii) $\frac{\partial(\phi_2 - \phi_M)}{\partial\psi} > 0$, $\frac{\partial(\phi_3 - \phi_2)}{\partial\psi} > 0$, $\frac{\partial(\phi_2 - \phi_M)}{\partial\bar{l}} < 0$, $\frac{\partial(\phi_3 - \phi_2)}{\partial\bar{l}} < 0$. *The increase of fees in betas for beta greater than one becomes steeper when investors face tighter borrowing constraints, i.e., when the fraction* ψ *of strictly constrained investors increases, or when* $\bar{l}$, *the borrowing bound of less constrained investors, decreases.*

(iii) if manager net performance is defined as $\alpha_i = R_i - \beta R_M - \phi_i$, *then* $\alpha_2 < \alpha_M$ *and* $\alpha_3 < \alpha_2$. *The manager's net performance is strictly decreasing in beta for betas greater than one.*

The intuition behind Proposition 4 is straightforward, and can been seen in Figure 3 which depicts the relation between betas and fees. We calibrate the model for multiple scenarios using the parameter values from Table 1. The blue line presents baseline relationship. Investors with low enough risk aversion choose the asset manager with $\beta_3 = 1.7$. Since these investors have higher willingness to pay for embedded leverage, they pay the highest fee in equilibrium. The next group of investors choose the asset manager $\beta_2 = 1.3$ and pay a lower fee. The most risk averse investors invest in the market index fund with $\beta_M = 1$.

The yellow line presents the setting with tighter leverage constraints. In this case, the willingness to pay for embedded leverage increase across investors and all the asset managers with betas above one can charge higher fees for the same beta. As a result, the scenario of tighter borrowing constraints features the increasing slope of fees in betas. The green line presents our final setting with a larger number of funds, for which we solve numerically for the equilibrium. As can be seen, the relation between beta and fees remains the same.

Finally, our results have a direct implication for the fund net performance as measured by the CAPM net-of-fee alpha. In particular, gross alphas for all funds equal to zero by definition as the stock market CAPM holds for our baseline model. As a result, each fund's net alpha equals minus the fee. Since fund fee is increasing in fund beta when beta is larger than one, net alpha must be decreasing in beta. Intuitively, investors pay for provision of leverage and the value of this service is not captured by the manager's net alpha.

[8] A sufficient condition is $\underline{\Gamma}\sigma_M^2 < \mu_M/\beta_3$. In our benchmark calibration (see Table 1), it is $\underline{\Gamma}\sigma_M^2 = 0.01$ and $\mu_M/\beta_3 = 0.0294$. The expected excess market return μ_M would have to be lower than 2% per year for this sufficient condition not to be fulfilled.

2.4 Testable Hypotheses

In our empirical work below we test our theory in the cross-section of the U.S. open-end equity mutual funds. We examine three specific hypotheses that are implied by Proposition 4. We start by formulating our hypothesis regarding the baseline cross-sectional relation between beta and fees.

Hypothesis 1. *After controlling for the known determinants of fees, fees increase with beta in the cross-section of funds for beta larger than one, and fees are flat in beta for beta smaller than one.*

Hypothesis 1 follows directly from Proposition 4(i). Importantly, we do not argue that fund beta is the only determinant of fund fees. Other known variables such as fund gross performance, its size, age, and fund family pricing policies may also affect fees. Our theory focuses on the effects of embedded leverage on fees and therefore complementary to the effects of other known determinants of fees. Consequently, in our empirical work we have to include a proper set of control variables to test whether the effect of beta is unique and is not being subsumed by other variables known to explain fees.

Since our model implies that leverage constraints drive the relation between beta and fees, it is natural to explore how the relation varies with the tightness of leverage constraints. This motivates the second hypothesis.

Hypothesis 2. *The cross–sectional relation between beta and fees for betas larger than one is stronger*

(1) for retail investors than for institutional investors,

(2) for funds which are introduced to the market during periods of tight borrowing constraints relative to funds introduced in less constrained periods.

Hypothesis 2 follows from Proposition 4(ii). The proposition suggests that the relation between beta and fees becomes stronger when either the fraction of strictly constrained investors increases, or when less constrained investors face a lower borrowing limit. Following Frazzini and Pedersen (2014), we suppose that retail investors face more severe leverage constraints relative to institutional investors and explore the cross-section of investor types. Following the proposition, we can think about this hypothesis in two ways. First, retail investors as a group can have a higher fraction of individuals who are severely constrained. Second, the borrowing limit of less constrained retail investors can be lower than the borrowing limit of less constrained institutional investors.

The second part of Hypothesis 2 is also implied by Proposition 4(ii). Previous work has shown that the tightness of leverage constraints varies not only in the cross-section of investors but also over time. If either the fraction of constrained investors or the borrowing limit varies over time, then the strength of the relation between beta and fees should vary over time. While our model considers a static setting which does not directly generate predictions regarding time variation in beta and fees within a given fund, it still has an implication for the cross-section of funds that is created in different time periods. In particular, the cross-section of funds introduced to the market in times of tight leverage constraints should have a stronger relation between fees and beta relative to the cross-section of funds introduced in times of weak leverage constraints. In our empirical work, we focus on specific time-varying measures of leverage constraints to test this hypothesis.

Finally, since the leverage-based theory suggests that funds with higher betas charge higher fees, it has a direct implication for fund net performance.

Hypothesis 3. *When betas are larger than one, fund net alpha declines in beta faster than gross alpha.*

Hypothesis 3 follows from Proposition 4(iii). As fees increase in fund beta for betas higher than one, our theory suggests fund net alphas should decline with beta due to the effect of fees. Importantly, we do not argue that fees are the only driver of the relation between beta and net alpha. For example, Frazzini and Pedersen (2014) suggest that stocks with high beta have low alpha. As a result, funds with higher beta can have a lower gross alpha and therefore lower net alpha. However, our model suggests that fees can further reduce net alphas of high beta funds beyond what is already implied by Frazzini and Pedersen (2014). As a result, when beta increases, fees progressively increase the gap between net and gross performance. Consequently, if we sort funds into portfolios with respect to their betas, we expect net alphas to decline in betas faster than gross alphas.

3 Data and Methodology

3.1 Data and Variables

We obtain our data from the CRSP U.S. Mutual Fund Database for the period from January 1991 to December 2016 since monthly reporting of fees, total net assets, and investment objectives becomes more precise and consistent after 1990. We start with the initial sample of all open-end mutual funds and keep only domestic equity funds using the information on investment objectives. We identify passive funds and exchange-traded funds (ETF) based on the CRSP definitions. To obtain a proper estimate of fund ownership costs to investors, we combine the information on fund annual expense ratios and loads. We follow Sirri and Tufano (1998) and Gil-Bazo and Ruiz-Verdú (2009) assuming the average fund share holding period of seven years. As a result, we define mutual fund total annual fee as a sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load.

We use three different datasets in our tests: fund share class dataset, fund-level dataset and fund launch dataset. We obtain the baseline, fund share class dataset directly from the CRSP database. To obtain fund-level dataset, we calculate averages of the CRSP variable across the share classes within the fund for each month, weighted by the share class total net assets in this month. The third dataset includes fund share class information only at the month of fund launch. To construct this dataset, we check the month of the fund's first appearance in the CRSP database, define this month as a month of fund launch and collect the information on this fund only over the given month. As a result, this dataset dataset includes only the fund share class which appeared first in the CRSP database, and only in the month of its first appearance.

3.2 Estimation of Market Beta and Fund Performance

We estimate the market model with a rolling window to evaluate fund market beta and its performance relatively to the market each month. Specifically, we estimate the following time-series regression for each fund:

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{Mt} - R_{ft}) + e_{it}. \tag{10}$$

In this regression, R_{it} is the return on fund i for month t, R_{ft} is the 1-month U.S. Treasury bill rate, R_{Mt} is the market return obtained from Kenneth French's website, and α_i is the average return unexplained by the market model that we further refer to as an estimate of fund CAPM alpha. We use two variants of this model. The first variant uses fund net returns to estimate net fund alpha and the second variant uses fund gross returns to estimate fund gross alpha. We define the monthly fund gross return as a sum of the monthly fund net return and one-twelfth of the annual fund fee. We refer to the estimate of β_i as the fund market beta. We present our results based on the estimates of fund betas derived from fund gross returns, but they remain virtually unchanged if we use betas derived from fund net returns. To estimate the models of fund performance, we require the fund to have at least 48 months of performance data available in the last 5 years and we use 5-year rolling regressions to obtain estimates for each month. For the fund launch sample, we estimate the models for the first 48 months of fund operation.

We construct two measures of fund competition across different levels of fund beta. The first measure counts the number of funds with the value of beta falling into each 0.1 bin (e.g., betas with value between 0.8-0.9 are in a bin, and betas between 1.1-1.2 are in another bin, etc.) in a specific month. The second measure is an asset-weighted Herfindahl-Hirschman Index (HHI) that is calculated for each 0.1 bin of beta in each month. Finally, we drop funds with extremely high fees in the sample (those above 99.9% of the sample distribution) and focus on betas in the middle 95% (i.e. 2.5%-97.5%) of the sample distribution.[9]

3.3 Summary Statistics

Table 2 presents the summary statistics for our variables. Panel A shows the information at the fund level. We observe that the average annual fee over the sample period is 1.38% and its standard deviation is 0.69%. The standard deviation of fees within a given fund over time is only 0.07%, indicating that most of the variation in fees is driven by differences across funds rather than time variation within funds. The distribution of fees is relatively symmetric across funds since the median fee is equal to 1.30%. The funds at the top 5% of the fee distribution charge a 2.54% fee while the funds at the bottom 5% of the distribution charge only 0.27%.

The average fund market beta is 1, with a standard deviation of 0.24 and within-fund standard deviation of 0.05. Similar to fund fees, there is more variation in market beta across funds than within funds. The market betas vary from 0.56 at the bottom 5% of the distribution to 1.39 at the top 5% of the distribution. The average gross CAPM alpha equals to 1.45% but it is statistically indistinguishable from zero since performance varies substantially in the sample, especially across funds. The average net alpha equals to 0.04% and is also statistically indistinguishable from zero. The difference between gross and net performance almost equals to fees suggesting that high fees of actively managed funds drive the average net performance towards zero. We can also see that passive funds represent 10% of the sample and ETFs represents 5% of the sample.[10]

[9] Our results are robust under different data cleaning criteria that drop more (e.g. those above 99%) or fewer (e.g. those above 99.99%) extremely high fees, or that focus on a narrower (e.g. the middle 90%) or wider (e.g. the middle 98%) range of betas.

[10] The definitions of passive fund and ETF do not necessarily overlap. The fund can be defined both as a passive fund and an ETF as in the example of any index-linked ETF. Index mutual funds meet the definition of passive funds but not of ETFs. In addition, some ETFs do not follow any index and therefore are considered to be actively managed.

Panel B presents the summary statistics for fund share classes and Panel C shows the information for the first share class in the month of fund launch. We can see that the average fee equals to 1.57% and it is slightly higher relatively to the fund level data since high fee share classes tend to have less assets under management relative to low fee share classes. The distributions of beta and alpha are very similar to the fund level data. In the fund share class sample, the average gross alpha is 1.33% while the average net alpha is -0.26% suggesting that returns to investors turn negative due to fees. The alphas are roughly the same in the sample of fund launches.

We next ask what drives variation in fees. On the one hand, mutual funds can frequently adjust fees due to time variation in fund beta or other fund characteristics. On the other hand, fees can be primarily driven by persistent, time-invariant differences across funds such as unobserved investor clienteles or if observed fund characteristics do not vary that much across time. The preliminary comparison of the standard deviation across observations to the standard deviation within funds has already revealed that fees and beta vary more across funds than within funds.

The last column of Panel A and Panel B presents the R-squared from the regression of variables on fund fixed effects and fund share class fixed effects respectively. We observe that time-invariant characteristics drive 96% of the variation in fees for the share class sample and 90% of variation in fees in the fund sample. We can also see that time-invariant characteristics drive 70% of the variation in betas in the share class sample and 66% of the variation in betas in the fund-level sample. The evidence suggests that almost all the variation in fees and most of the variation in betas is driven by differences across funds.

These observations have two key implications for our analysis. First, we will primarily focus on cross-sectional tests rather than on time-series tests since almost all the variation in fees is cross-sectional. Second, the evidence on the lack of time-series variation is consistent with our model which is myopic and emphasizes the differences across funds rather than within funds.

4 Testing Hypotheses

4.1 Relation between Market Beta and Fund Fees

4.1.1 Main Tests

We start by testing the first hypothesis implied by our model: fund fees increase in beta when beta is larger than one and fees are flat in beta when beta is smaller than one. Our baseline econometric specification is:

$$Fee_{ift} = \gamma_f + \gamma_t + \lambda Beta_{ift} + \rho X_{ift} + e_{ift}. \tag{11}$$

In this regression Fee_{ift} is a fee for fund i in fund family f in month t, $Beta_{ift}$ is the fund market beta, γ_f is a fund family fixed effect, γ_t is a month fixed effect and X_{ift} is a set of fund-level time varying control variables such as the fund CAPM alpha, the logarithm of fund age in months, the logarithm of fund total net assets, a dummy variable that equals one if a fund is passively managed and a dummy variable that equals one if a fund is an ETF. The standard errors are double-clustered by fund family and month. We use fund-months as a unit of observation in the fund-level tests and fund share class-months as a unit of observation in the fund share class tests. Our specifications exploit cross-sectional variation among funds within a given family.

We first examine the relationship between beta and fees non-parametrically. Figure 4 presents the binscatter plot of residual fees and beta separately for betas larger than one and smaller than one. In these plots, the residual fee is estimated by two steps. First, we regress the fee on all the control variables and fixed effects as specified in regression (13), and then we calculate the residual as the original fee minus the predicted value based on the estimation in the first step. We can see that the figure confirms the model central prediction. In particular, for beta larger than one fees increase with market beta while for beta smaller than one fees are flat in beta.

Table 3 presents the regression results confirming the relation presented in Figure 4 and supporting the first model prediction. Columns (1) and (2) present the estimates based on fund-level regressions for betas smaller than one while columns (3) and (4) present the estimates for betas larger than one. Column (1) shows that the estimate of the coefficient on beta for betas below one is both economically and statistically insignificant. Column (2) confirms that this result does not change when we control for fund performance. Columns (3) and (4) show that fees increase in beta when beta is larger than one. This relationship is statistically significant at the 1% level. The relation between betas and fees for betas above one is economically meaningful: when fund beta increases from 1 to 1.7, the 99th percentile of beta distribution, fund fees increase by 25 basis points. This relationship also stands as economically significant relative to the effects of other determinants of fund fees. For example, a one-standard deviation increase in fund beta is associated with an increase in fees of 8 basis points. At the same time, an increase of one standard deviation in log fund size is associated with a reduction of 11 basis points in fees, while an increase of one standard deviation in log fund age is associated with an increase of 3 basis points in fees.

Columns (5)-(8) repeat the specification from columns (1)-(4) using the share class level data. We observe that the estimates are very similar to those obtained through the fund-level data. Column (5) shows that effects of beta on fees for betas smaller than one is negative and significant at the 10% level, but when we control for fund performance in column (6), both the statistical and the economic significance disappear. Columns (7) and (8) show that for betas larger than one, the coefficients on beta are large and significant. These coefficients exhibit even larger economic magnitudes relative to the fund-level regressions: when fund beta increases from 1 to 1.7, the 99th percentile of beta distribution, fund fees increase by 31 basis points.

In sum, the evidence supports the leverage-driven relation between fees and beta since fees increase in beta only for betas larger than one, as predicted by the model.

4.1.2 Robustness to Inclusion of Competition Measures

We next examine the robustness of our main results to the inclusion of various measures of competition in our econometric specification. Our model argues that the willingness to pay for embedded leverage drives the relation between fund fees and fund market beta for betas larger than one. Alternatively, our empirical findings could also be driven by the effects of competition on fund fees. If the competition among funds weakens with beta for this range of betas, fee could increase with beta due to the effects of weaker competition and not due to the effects of the demand for leverage. Figure 5 presents the distribution of the number of funds across market betas and provides some basis for this potential concern. In particular, when fund beta increases at the range of betas larger than one, the fraction of funds with this beta steadily declines.

We repeat the specifications for larger than one betas including the two measures of competition to test the robustness of our results to the competition-based explanation. Table 4 presents the estimation results. Columns (1) and (2) present the results for the fund-level data and columns (3) and (4) show the results for share-class level data. We can see that controlling for fund competition does not affect our results. In particular, the estimates of the coefficients on beta are quantitatively and qualitatively similar to the estimates from Table 3. The combined evidence suggests that our results are not driven by the relationship between fund beta and competition among funds.

4.1.3 Time Series Tests

We continue and examine the relation between beta and fees in the time series. Our main tests asked whether fees vary with beta across funds within a fund family. Table 2 provides a clear motivation for these cross-sectional tests showing that most of the variation in both fees and beta comes from time-invariant heterogeneity across funds. Our next series of tests ask whether changes in betas across time within funds are associated with changes in fund fees. To implement the time series tests, we replace fund family fixed effects with fund (or fund share class) fixed effects in our regression specifications.

Table 5 presents the estimation results. Columns (1)-(4) present the specifications based on the fund-level data and columns (5)-(8) show the specifications based on the share class data. For betas smaller than one the effects remain small and insignificant as in the main tests. For betas larger than one the positive relation between beta and fees is recovered, but it is only weakly statistically significant and economically small. These results suggest that the relationship between beta and fees is mainly cross-sectional and the changes in betas within funds are only weakly related to the changes in fund fees.

4.2 Variation in Borrowing Constraints

4.2.1 Comparison of Retail Investors and Institutional Investors

Having tested the main hypothesis, we next examine the second prediction and explore the effect of variation in tightness of borrowing constraints. We first focus on the tests across investor types. The model predicts that the relation between beta and fees for betas larger than one is expected to be stronger among retail investors relative to institutional investors. This implication is based on the intuition that retail investors are more likely to face borrowing constraints (Frazzini and Pedersen 2014). We test the prediction by introducing two dummy variables: a dummy variable that equals one if a share class is offered to retail investors, and a dummy variable that equals one if a share class is offered to institutional investors. We add these dummies to our main specification and interact them with market beta to evaluate the relation between beta and fees for different investor clienteles.

Table 6 presents the results of the analysis. Columns (1) and (2) present the estimates when we use the share class level dataset. We can see that the coefficient on the interaction between market beta and the "retail" dummy is 0.44 while the coefficient on the interaction between market beta and the "institutional" dummy equals to 0.30. This result indicates that the effect of beta on fees is almost 50% larger for retail investors. Column (2) shows that the result holds when we control for fund performance. We also formally test the significance of the difference between the coefficients and present the p-values at the bottom of Table 6. The difference between the coefficients is statistically significant at the 5% level.

Columns (3) and (4) present an additional series of tests when we investigate a significantly smaller sample of funds at launch, when a share class was first offered to the investors. We can see that the coefficient on the interaction between market beta and the "retail" dummy equals to 0.44 while the coefficient on the interaction between market beta and the "institutional" dummy equals to 0.22. This result implies that when a share class is offered to retail investors, the fund family charges them almost twice more for the same beta relative to institutional investors. The difference between the coefficients for the fund launch sample is statistically significant with a p-value of 2%.

In sum, our cross-sectional evidence confirms the second prediction of the model since more constrained retail investors pay more for beta relative to less constrained institutional investors.

4.2.2 Time Variation in Tightness of Borrowing Constraints

We continue and examine the second prediction of the model using the time variation in borrowing constraints. The model predicts that the relation between beta and fees for betas larger than one is more pronounced in times when it is more difficult to borrow capital. We use three measures of borrowing constraint tightness: the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014), the intermediary capital ratio (ICR) from He, Kelly, and Manela (2017), as well as the leverage constraint tightness (LCT) measure from Boguth and Simutin (2018). We use monthly variation in each measure and define periods when a measure takes on extreme values as periods of tight borrowing constraints. We refer to these periods as "constrained" periods. Low values of BAB and ICR indicate tighter borrowing constraints. Consequently, we define periods with BAB and ICR in the first quartile of their distributions across time as constrained periods. High values of LCT indicate tighter borrowing constraints. As a result, we define periods with LCT in the fourth quartile of its distribution as constrained periods.

We next introduce two dummy variables: a dummy variable that equals one if a period is defined as constrained according to one of the measures, and a dummy variable that equals one if a period is defined as unconstrained. We add these dummies to our main specification and interact them with market beta to evaluate the relation between fee and beta in times when it is more difficult to borrow capital relative to other time periods. Since our theory focuses on cross-sectional differences between asset managers, in this series of tests we analyze the sample of funds at launch. In particular, we ask whether the cross-section of funds introduced in constrained periods exhibits higher fees per unit of beta over this period relative to the cross-section of funds introduced in less constrained periods.

Table 7 presents the results of this analysis. In these tests, we focus on extreme values of borrowing constraints tightness and draw our samples from the first and the fourth quartiles of each measure. Columns (1) and (2) show the results when we use the BAB factor as a measure of borrowing constraint tightness. We observe that the coefficient on the interaction between market beta and a "constrained" dummy equals to 0.54 and it is statistically significant at the 1% level. At the same time, the coefficient on the interaction between market beta and an "unconstrained" dummy equals to 0.12 and it is not statistically significant. The p-value of the t-test for the difference between the coefficients equals 4%. These results suggest that funds introduced in constrained periods as measured by the BAB factor, charge four times more per unit of beta relative to funds introduced in unconstrained periods.

Columns (3) and (4) show the results for the ICR measure and are similar to the results obtained in columns (1) and (2). The coefficient on the interaction between market beta and a "constrained" dummy equals to 0.61 and the coefficient on the interaction between market beta and an "unconstrained" dummy equals to 0.24. The p-value of the t-test for the difference between the coefficients equals 4%. These results indicate that the funds introduced in constrained periods charge three times more per each unit of beta.

Finally, we repeat the analysis using the LCT measure in columns (5) and (6). The results show that the coefficient on the interaction between market beta and a "constrained" dummy is twice larger than the coefficient on the interaction between market beta and an "unconstrained" dummy. However, the difference between the coefficients is not statistically significant.

In sum, the evidence from the time variation in borrowing constraints additionally confirms the second hypothesis of the model since in constrained periods investors pay more for the same beta relatively to unconstrained periods.

4.3 Implications for Fund Net Performance

We next examine the last hypothesis and study the effects of leverage constraints on fund net performance. The model's third prediction suggests that leverage constraints impact fund net performance as measured by net CAPM alpha since investors pay fees for embedded leverage measured by fund market beta. In particular, the model predicts that fund net alpha declines in beta faster than its gross alpha.

We conduct a portfolio analysis to test this prediction. We sort funds with betas larger than one into five equally-weighted portfolios according to the fund's beta and calculate mean gross and net alphas as well as mean gross and net returns for these portfolios.[11] Panel A in Table 8 presents the results for the share class level dataset. In column (1) we observe that fees are steadily increasing across portfolios as predicted by our model. The difference in fees between the high beta portfolio and the low beta portfolio equals to 0.23%. Column (2) shows the average gross CAPM alphas across the portfolios. We can see that the gross alpha is declining with beta as predicted by Frazzini and Pedersen (2014). The difference in gross performance between the high beta portfolio and the low beta portfolio equals to -0.37% but it is not statistically significant even at the 10% level. Column (3) presents the average net alpha such that the difference between column (3) and column (2) roughly equals to fund fees presented in column (1). We observe that fund net performance declines much faster with beta than fund gross performance. The difference in net performance between the high beta portfolio and the low beta portfolio equals to -0.60% and it is statistically significant at the 5% level. As predicted by the model, the increase in fees drives the decline in net performance. The difference in fees almost doubles the difference in gross performance between the high beta portfolio and the low beta portfolio. The results are very similar for the fund-level analysis which is presented in Panel B.

These results indicate that two effects can jointly explain why net performance declines with beta. The first effect is the increase in fees that is suggested by our theory and has been confirmed by the evidence that we presented earlier. The second effect is a reduction in gross alphas suggested by Frazzini and Pedersen (2014). While high beta funds significantly underperform net of fees, columns (4) and (5) show that these funds tend to have higher average returns. This evidence is consistent with our model where investors willingly pay higher fees to lever up the market portfolio and obtain access to higher net returns.

Our results address the well-known evidence on poor performance of the average actively managed mutual fund. Specifically, a large body of literature shows that many funds charge

[11] We exclude passive mutual funds and ETFs from this analysis.

fees that are significantly higher than their risk-adjusted returns. Our model provides a novel explanation for this phenomenon which is supported by the data. In our framework, high beta funds provide embedded leverage to investors on top of risk-adjusted performance and subsequently charge higher fees. As a result, net alpha as a sole measure of fund performance does not capture the benefits of embedded leverage as an additional service provided by asset managers to their investors.

5 Conclusion

In this article we present a new model suggesting that investors pay additional fees for embedded leverage provided by high beta asset managers. If investors face leverage constraints and cannot lever up the market portfolio on their own, they can approach asset managers to obtain access to leveraged returns. Our theory predicts that asset management fees increase in beta when beta is larger than one and that this relationship is more pronounced when leverage constraints tighten up both across investors and across time. We provide empirical evidence that supports these predictions.

Our results imply that high beta funds provide an additional service to investors who face leverage constraints. These funds can charge higher fees for embedded leverage and investor can willingly pay high fees even at the absence of higher risk-adjusted performance. The fees vary across investors and market conditions in a manner consistent with the leverage-based explanation. These results suggest that risk-adjusted performance alone does not fully describe the whole range of services provided by asset managers to their investors and that the provision of embedded leverage is one such service. Consequently, high beta funds might appear as "underperforming" net of fees while their investors can actually be better off due to gaining access to leverage.

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Figure 1: Willingness to Pay, Risk Aversion and Fund Beta

This figure presents constellations of investor i's risk aversion γ_i and fund j's fee ϕ_j for which j is preferred over the market ETF with $\beta_M = 1$ and $\phi_M = 0$. The blue region considers a fund with $\beta_j = 1.1$, the yellow region considers a fund with $\beta_j = 1.3$. We set $\mu_M = 0.05$ and $\sigma_M = 0.2$ (see also Table 1). The left plot stands for investors who face strict borrowing constraints, the right plot stands for less constrained investors with $l = 2$. The dashed lines stand for the ϕ_j value above which condition (4) is fulfilled, i.e., in which the region is linear.

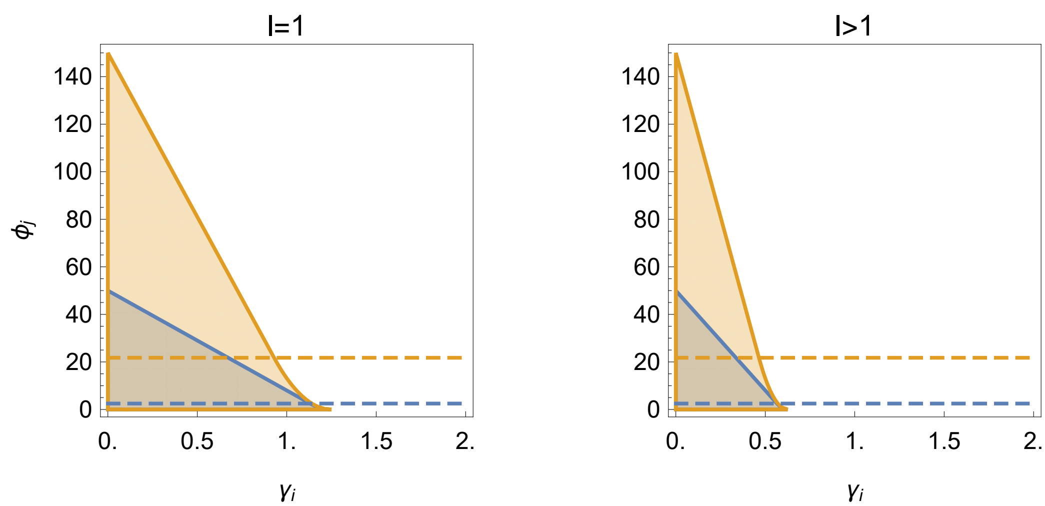

Figure 2: Distribution of Investors across Funds

This figure illustrates how investors sort into asset managers based on their risk aversion γ_i, given four managers with $\beta > 1$, the market ETF with $\beta_1 = \beta_M = 1$, and possible additional funds with $\beta < 1$. Further parameters are given by Table 1. Fund fees are set exemplary to $(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5) = (0, 2.5, 25, 65, 120)$ basis points. All the investors face strict borrowing constraints ($l = 1$).

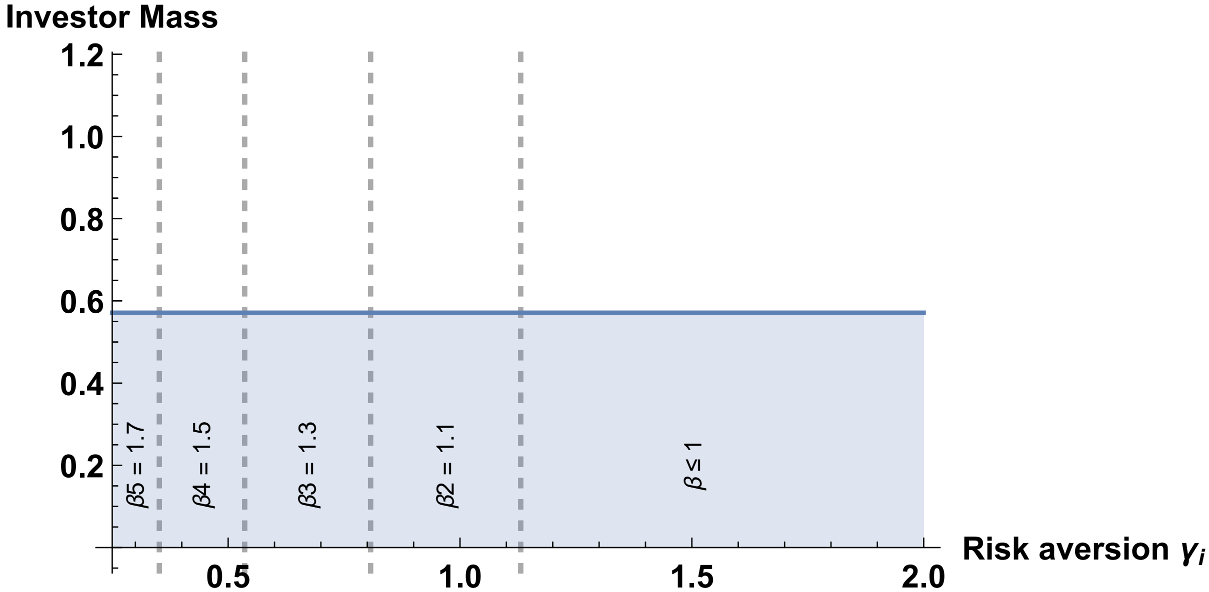

Figure 3: The Theoretical Relationship between Betas and Fees

This figure presents the relation between fund betas and fees as suggested by our model. The blue line stands for a scenario with "few" funds (i.e., two $\beta > 1$-funds, the market ETF, and an arbitrary number of $\beta < 1$-funds) where the parameters are set according to the "less constrained" scenario in Table 1. The yellow line repeats a scenario with "few" funds but for the case that all the investors face strict borrowing constraints ($l = 1$). The green line describes a setting with "many" funds according to Table 1 for which we numerically solve for the equilibrium. In all the scenarios, fund fees result endogenously from the model equilibrium.

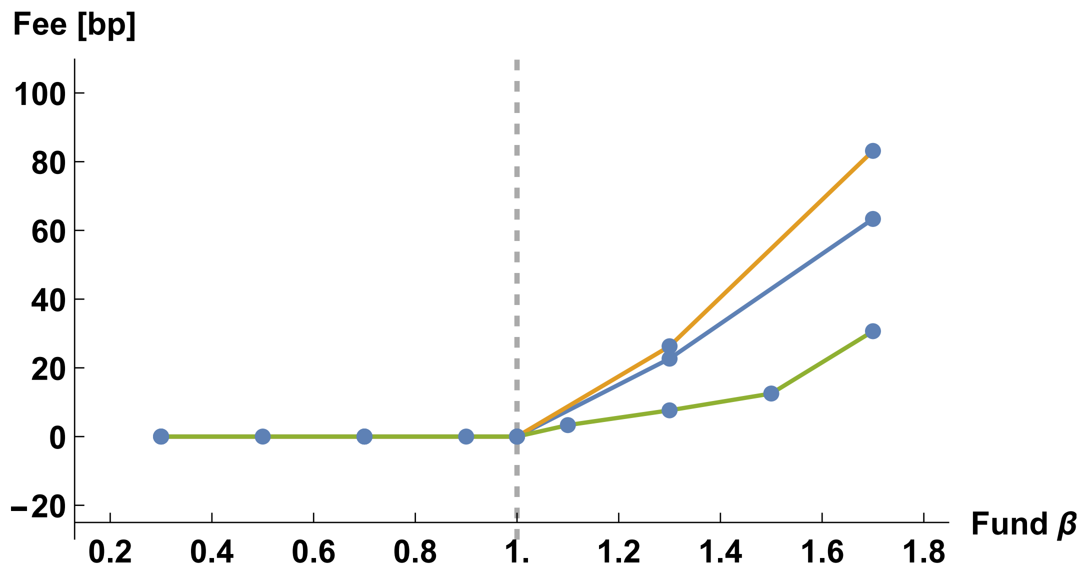

Figure 4: The Empirical Relationship between Beta and Fees

This figure presents the binscatter plot of residual fees and beta separately for betas larger than one and smaller than one. In these plots, the residual fee is estimated by two steps. First, we regress the fee on all the control variables and fixed effects. Second, we calculate the residual as the original fee minus the predicted value based on the estimation in the first step.

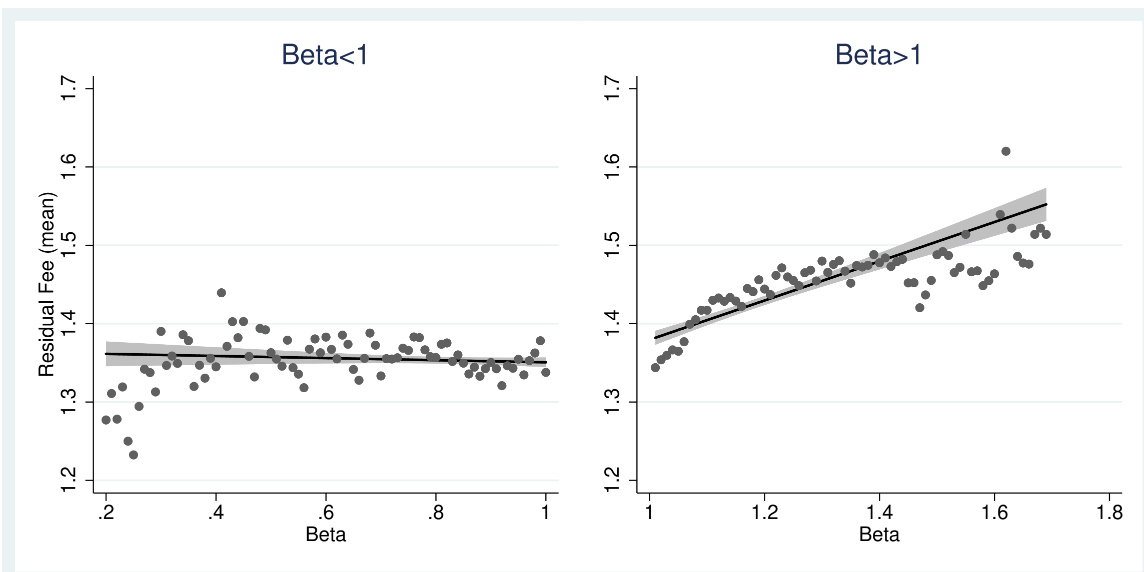

Figure 5: Distribution of Fund Beta

This figure presents the empirical distribution of funds across market betas. The y-axis presents the fraction of funds for each level of beta. The market betas are estimated using the market model as described in Section 3.

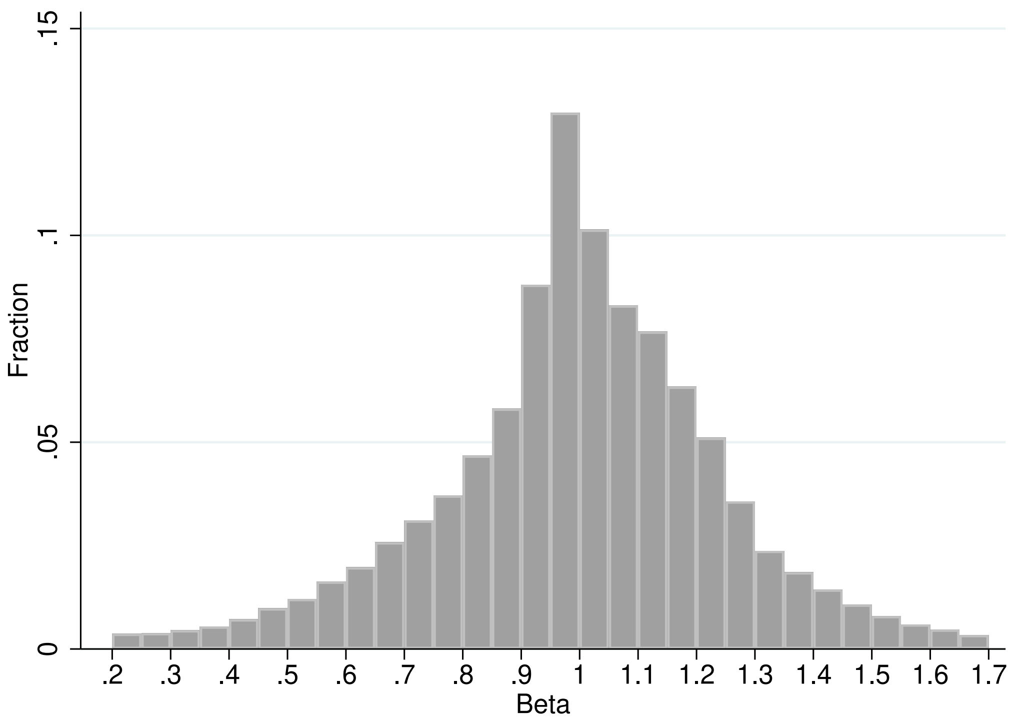

Table 1: Model Parameters, Borrowing Constraints Scenarios and Fund Betas

This table presents the parameters we use to calibrate the model for different scenarios.

Parameter		Value	
Stock market			
Expected stock market return	μ_M	0.05	
Stock market volatility	σ_M	0.2	
Fund investors			
Highest risk aversion	$\overline{\Gamma}$	2	
Lowest risk aversion	$\underline{\Gamma}$	0.25	
		More constrained	Less constrained
Proportion of strictly borrowing constr. investors ($l = 1$)	ψ	1	0.25
Bound for less borrowing constrained investors	$\bar{l}$	—	2
Fund characteristics			
		Many funds	Few funds
	β_1	1.0	1.0
	β_2	1.1	1.3
Fund betas	β_3	1.3	1.7
	β_4	1.5	—
	β_5	1.7	—

Table 2: Summary Statistics

This table presents summary statistics for the sample of fund-month observations over the period of 1991-2016. Panel A presents the statistics at the fund level, Panel B presents the statistics at the fund share class level and Panel C presents the statistics at the fund share class level at the time of fund launch. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Market beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* and *Net CAPM alpha* are estimates of the intercept from the market models for fund gross returns and fund net returns. *Fund TNA, Fund Age, Passive fund* indicator, *ETF* indicator and *Retail fund* indicator are from the CSRP mutual fund database. *Number of funds per beta bin* is the number of funds with the value of beta falling into each 0.1 bin (e.g., betas with value between 0.8-0.9 are in a bin, and betas between 1.1-1.2 are in another bin, etc.) in a specific month. *HHI per beta bin* is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. The last columns of Panels A and B report the R^2 of the regressions of variables on fund fixed effects or fund share class fixed effects.

Panel A: Funds	N	Mean	SD	Within SD	5%	25%	50%	75%	95%	R^2 - fund FE
Fee (%)	439,539	1.38	0.69	0.07	0.27	0.93	1.30	1.84	2.54	0.90
Market beta	439,539	1.00	0.24	0.05	0.56	0.87	1.00	1.14	1.39	0.66
Gross CAPM alpha (%)	439,539	1.45	5.52	1.88	-5.57	-1.14	0.95	3.60	10.72	0.34
Net CAPM alpha (%)	439,539	0.04	5.47	1.87	-7.16	-2.46	-0.34	2.12	9.17	0.34
Log(TNA)	439,539	5.64	1.86	0.37	2.54	4.37	5.68	6.95	8.65	0.85
Log(Age)	439,539	4.84	0.47	-	4.13	4.48	4.84	5.18	5.64	-
(0,1) Passive fund	439,539	0.10	0.30	-	0.00	0.00	0.00	0.00	1.00	-
(0,1) ETF	439,539	0.05	0.22	-	0.00	0.00	0.00	0.00	1.00	-
N of funds per beta bin	439,539	0.37	0.22	-	0.05	0.17	0.31	0.28	0.69	
HHI per beta bin	439,539	0.05	0.05	-	0.01	0.03	0.04	0.05	0.12	

Panel B: Fund Share Classes	N	Mean	SD	Within SD	5%	25%	50%	75%	95%	R^2 - fund share class FE
Fee (%)	989,552	1.57	0.75	0.03	0.37	0.99	1.52	2.15	2.76	0.96
Market beta	989,552	1.00	0.21	0.04	0.61	0.90	1.00	1.13	1.35	0.70
Gross CAPM alpha (%)	989,552	1.33	4.95	0.12	-4.88	-1.12	0.85	3.27	9.59	0.39
Net CAPM alpha (%)	989,552	-0.26	4.90	0.13	-6.63	-2.62	-0.63	1.63	7.81	0.39
Log(TNA)	989,552	4.15	2.34	0.36	0.09	2.65	4.27	5.80	7.76	0.88
Log(Age)	989,552	4.81	0.44	-	4.15	4.46	4.77	.12	5.56	-
(0,1) Passive fund	989,552	0.07	0.26	-	0.00	0.00	0.00	0.00	1.00	-
(0,1) ETF	989,552	0.02	0.15	-	0.00	0.00	0.00	0.00	1.00	-
(0,1) Retail fund	989,552	0.66	0.47	-	0.00	0.00	1.00	1.00	1.00	-
N of funds per beta bin	989,552	1.14	0.69	-	0.15	0.49	1.13	1.84	2.11	
HHI per beta bin	989,552	0.03	0.03	-	0.01	0.14	0.18	0.03	0.07	

Table 2: Summary Statistics (Continued)

This table presents summary statistics for the sample of fund-month observations over the period of 1991-2016. Panel A presents the statistics at the fund level, Panel B presents the statistics at the fund share class level and Panel C presents the statistics at the fund level at the time of fund launch. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Market beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* and *Net CAPM alpha* are estimates of the intercept from the market models for fund gross returns and fund net returns. *Fund TNA, Fund Age, Passive fund* indicator, *ETF* indicator and *Retail fund* indicator are from the CSRP mutual fund database. *Number of funds per beta bin* is the number of funds with the value of beta falling into each 0.1 bin (e.g., betas with value between 0.8-0.9 are in a bin, and betas between 1.1-1.2 are in another bin, etc.) in a specific month. *HHI per beta bin* is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. The last columns of Panels A and B report the R^2 of the regressions of variables on fund fixed effects or fund share class fixed effects.

Panel C: Funds at Launch	N	Mean	SD	5%	25%	50%	75%	95%
Fee (%)	11,520	1.60	0.83	0.29	0.98	1.50	2.22	2.94
Market beta	11,520	0.98	0.25	0.52	0.85	0.99	1.12	1.41
Gross CAPM alpha (%)	11,520	1.32	6.05	-6.22	-1.46	0.80	3.62	11.56
Net CAPM alpha (%)	11,520	-0.27	6.00	-8.06	-2.89	-0.61	1.93	9.69
Log(TNA)	11,520	1.56	2.50	-2.30	-0.22	1.71	3.43	5.46
(0,1) Passive fund	11,520	0.08	0.26	0.00	0.00	0.00	0.00	1.00
(0,1) ETF	11,520	0.04	0.19	0.00	0.00	0.00	0.00	1.00
(0,1) Retail fund	11,520	0.59	0.49	0.00	0.00	1.00	1.00	1.00

Table 3: Regressions of Mutual Fund Fees on Market Beta

This table presents the results from regressing fund fees on fund beta and fund characteristics separately for funds with betas larger than one and funds with betas smaller than one. Columns (1)-(4) present the specifications at the fund level. Columns (1) and (2) present the specifications for funds with beta less than one while columns (3) and (4) present the specifications for funds with beta larger than one. Columns (5)-(8) repeat the specifications from columns (1)-(4) at the fund share class level. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = Fee	Fund level				Share class level			
	beta<1		beta>1		beta<1		beta>1	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Market beta	-0.07	-0.02	0.33***	0.35***	-0.09*	0.01	0.44***	0.48***
	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)
Gross CAPM Alpha		0.01***		0.07***		0.17***		0.19***
		(0.00)		(0.02)		(0.02)		(0.02)
Log(age)	0.10***	0.10***	0.07**	0.07***	0.17***	0.17***	0.20***	0.21***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)
Log(TNA)	-0.04***	-0.04***	-0.06***	-0.06***	-0.06***	-0.06***	-0.08***	-0.09***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
(0,1) Passive	-0.60***	-0.60***	-0.51***	-0.52***	-0.60***	-0.60***	-0.51***	-0.52***
	(0.06)	(0.06)	(0.08)	(0.08)	(0.06)	(0.06)	(0.09)	(0.09)
(0,1) ETF	0.06	0.04	-0.35	-0.34	0.11	0.09	-0.27	-0.24
	(0.15)	(0.15)	(0.23)	(0.23)	(0.12)	(0.12)	(0.25)	(0.25)
Observations	218,872	218,872	219,912	219,912	476,797	476,797	511,810	511,810
R-squared	0.66	0.66	0.69	0.69	0.46	0.47	0.47	0.48
Fund family fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 4: Regressions of Mutual Fund Fees on Market Beta and Measures of Competition

This table presents the results from regressing fund fees on fund beta, controlling for measures of competition and fund characteristics. All regressions are estimated for funds with betas larger than one. Columns (1) and (2) present the specifications at the fund level while columns (3) and (4) repeat the specifications from columns (1) and (2) at the fund-share class level. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = Fee	Fund level		Share class level	
	(1)	(2)	(3)	(4)
Market beta	0.28***	0.38***	0.33***	0.52***
	(0.06)	(0.07)	(0.07)	(0.06)
N of funds per beta bin	-0.06		-0.04**	
	(0.05)		(0.02)	
HHI per beta bin		-0.18*		-0.29**
		(0.11)		(0.14)
Gross CAPM Alpha	0.07***	0.07***	0.18***	0.18***
	(0.02)	(0.02)	(0.02)	(0.02)
Observations	219,912	219,912	511,810	511,810
R-squared	0.69	0.69	0.48	0.48
Control variables	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes

Table 5: Regressions of Mutual Fund Fees on Market Beta and Fund Fixed Effects

This table presents the results from regressing fund fees on fund beta and fund fixed effects separately for fund with betas larger than one and funds with betas smaller than one. Columns (1)-(4) present the specifications at the fund level using fund fixed effects. Columns (1) and (2) present the specifications for funds with beta less than one while columns (3) and (4) present the specifications for funds with betas larger than one. Columns (5)-(8) repeat the specifications from columns (1)-(4) at the fund share class level using fund share class fixed effects. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = Fee	Fund level				Share class level			
	beta<1		beta>1		beta<1		beta>1	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Market beta	0.01	0.05	0.07*	0.06	-0.00	-0.05	0.06**	0.05*
	(0.05)	(0.05)	(0.04)	(0.04)	(0.03)	(0.03)	(0.02)	(0.03)
Gross CAPM Alpha		-0.07**		-0.01		-0.06***		-0.02*
		(0.03)		(0.01)		(0.01)		(0.01)
Observations	218,809	218,809	218,862	218,862	476,570	476,570	511,625	511,625
R-squared	0.92	0.92	0.92	0.92	0.97	0.97	0.97	0.97
Fund fixed effects	Yes	Yes	Yes	Yes	No	No	No	No
Fund share class fixed effects	No	No	No	No	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 6: Regressions of Mutual Fund Fees on Market Beta and Investor Types

This table presents the results from regressing fund fees on market betas interacted with indicators for retail and institutional share classes. All regressions are estimated for funds with betas larger than one. Columns (1) and (2) present the specifications at the fund share class level while columns (3) and (4) repeat the specifications from columns (1) and (2) at the time of fund launch. P-values for tests of differences between the coefficients are reported. In these tests, the null hypothesis is that the coefficient on the interaction between market beta and an indicator for institutional share class equals to the coefficient on the interaction between market beta and an indicator for retail share class. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = Fee	Share Class		Fund Launch	
	(1)	(2)	(3)	(4)
(0,1) Institutional * Market Beta	0.30***	0.34**	0.22***	0.24***
	(0.07)	(0.06)	(0.06)	(0.06)
(0,1) Retail * Market Beta	0.44***	0.46**	0.43***	0.44***
	(0.06)	(0.05)	(0.09)	(0.09)
(0,1) Retail	0.73**	0.73**	0.65***	0.65***
	(0.08)	(0.08)	(0.12)	(0.12)
Gross CAPM Alpha		0.12**		0.07***
		(0.02)		(0.02)
P-values for tests of differences between coefficients				
H_0: Institutional * Market Beta = Retail * Market Beta	0.05	0.06	0.02	0.02
Observations	476,797	476,797	5,186	5,186
R-squared	0.68	0.68	0.70	0.71
Control variables	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes

Table 7: Regressions of Mutual Fund Fees on Market Beta and Tightness of Borrowing Constraints

This table presents the results from regressing fund fees on market betas interacted with measures of tightness of borrowing constraints. All regressions are estimated for funds with betas larger than one and at the time of fund launch, and the sample is from the first and fourth quartiles of each borrowing constraint measures. Columns (1) and (2) present the specifications using BAB to measure tightness of borrowing constraints, columns (3) and (4) repeat the specifications using ICR, and columns (5) and (6) repeat the specifications using LCT. *Constrained* indicator equals one if BAB and ICR are in the first quartile of their distributions across time and if LCT is in the fourth quartile of its distribution across time. *Unconstrained* indicates the other quarter. P-values for tests of differences between the coefficients are reported. In these tests, the null hypothesis is that the coefficient on the interaction between market beta and an indicator for constrained time period equals to the coefficient on the interaction between market beta and an indicator for unconstrained time period. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = Fee						
Measure of Borrowing Constraints Tightness	BAB		ICR		LCT	
	(1)	(2)	(3)	(4)	(5)	(6)
(0,1) Constrained * Market Beta	0.54***	0.55***	0.61***	0.66***	0.31**	0.30**
	(0.13)	(0.13)	(0.15)	(0.16)	(0.13)	(0.14)
(0,1) Unconstrained* Market Beta	0.12	0.14	0.24***	0.22**	0.17	0.18
	(0.10)	(0.11)	(0.08)	(0.09)	(0.14)	(0.14)
Gross CAPM Alpha		0.12***		0.07*		0.05
		(0.02)		(0.04)		(0.03)
P-values for tests of differences between coefficients						
H_0: Constrained * Market Beta = Unconstrained * Market Beta	0.03	0.04	0.06	0.04	0.50	0.53
Observations	2,616	2,616	2,150	2,150	2,462	2,462
R-squared	0.72	0.72	0.74	0.74	0.70	0.70
Control variables	Yes	Yes	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

Table 8: Average Returns, CAPM Alphas and Fees for Mutual Fund Portfolios Sorted by Market Beta

This table presents average returns, CAPM alphas, and fees for five equally weighted mutual fund portfolios sorted by their market betas. All funds have betas larger than one. Panel A presents the results using fund share class data and Panel B presents the results using fund level data. Quintile (1) represents the portfolio of funds with the lowest betas while quintile (5) represents the portfolio of funds with the highest betas. The bottom panels present the difference in the averages between quintiles (1) and (5) as well as the t-statistics for tests of differences between the averages.

Panel A: Share Class Level					
	(1)	(2)	(3)	(4)	(5)
Quintile	Fee	Gross CAPM Alpha	Net CAPM Alpha	Gross Return	Net Return
(1) - Low Beta	1.65	0.74	-0.92	8.17	6.49
	(0.14)	(1.56)	(1.53)	(55.20)	(55.18)
(2)	1.71	0.80	-0.93	8.30	6.61
	(0.14)	(2.04)	(2.01)	(58.60)	(58.65)
(3)	1.77	0.73	-1.07	8.73	6.91
	(0.13)	(2.49)	(2.46)	(63.77)	(63.74)
(4)	1.81	0.44	-1.39	9.32	7.56
	(0.15)	(3.01)	(3.00)	(69.56)	(69.55)
(5) - High Beta	1.88	0.37	-1.52	9.93	8.02
	(0.15)	(3.80)	(3.79)	(77.91)	(77.91)
(5) - (1)	0.23	-0.37	-0.60	1.77	1.54
t-statistic	18.10	-1.47	-2.40	0.36	0.40
Panel B: Fund Level					
(1) - Low Beta	1.48	0.81	-0.67	8.22	6.75
	(0.17)	(1.66)	(1.56)	(55.41)	(55.26)
(2)	1.55	0.97	-0.61	8.54	7.00
	(0.18)	(2.11)	(2.09)	(58.70)	(58.73)
(3)	1.60	0.84	-0.80	8.90	7.29
	(0.15)	(2.61)	(2.59)	(63.57)	(63.52)
(4)	1.66	0.65	-1.03	9.55	7.75
	(0.21)	(3.20)	(3.18)	(69.82)	(69.59)
(5) - High Beta	1.70	0.40	-1.30	9.42	7.87
	(0.20)	(3.90)	(3.89)	(79.77)	(79.70)
(5) - (1)	0.22	-0.41	-0.63	1.20	1.12
t-statistic	12.83	-1.57	-2.42	0.20	0.19

Appendix

A Model: Proofs and Additional Results

We provide proofs for our model results that are not contained in the main text, as well as additional results for generalized cases.

Proof of Proposition 1 Condition 1 of the Proposition is derived in the main text. For condition 2, compare two different funds k and j with $\beta_k > \beta_j$. Let ω_i^{j*} be the optimal allocation for fund j, such that the related utility for investor i is

$$\omega_i^{j*}(\beta_j \mu_M - \phi_j) + R_f - \frac{\gamma_i}{2} \omega_i^{j*2} \beta_j^2 \sigma_M^2 \tag{12}$$

according to (1). We compare this to the utility that fund k provides, which is

$$\omega_i^k(\beta_k \mu_M - \phi_k) + R_f - \frac{\gamma_i}{2} \omega_i^{k2} \beta_k^2 \sigma_M^2. \tag{13}$$

Now choose the weight of the risky investment for fund k as $\omega_i^k = \omega_i^{j*} \frac{\beta_j}{\beta_k}$. Then we have $\omega_i^k < \omega_i^{j*}$ and the related utility is obtained as

$$\omega_i^{j*}(\beta_j \mu_M - \frac{\beta_j}{\beta_k} \phi_k) + R_f - \frac{\gamma_i}{2} \omega_i^{j*2} \beta_j^2 \sigma_M^2. \tag{14}$$

Comparing (12) and (14), we see that fund k dominates fund j unless the fees fulfill the condition $\phi_j \leq \frac{\beta_j}{\beta_k} \phi_k$. Therefore, funds with $\phi_j > \frac{\beta_j}{\beta_k} \phi_k$ and thus especially funds with $\phi_j > \phi_k$ for $\beta_j < \beta_k$ are dominated.

General version of Proposition 2 We characterize the investor fund preference dependent on their risk aversion in Proposition 2 and focus on the case that condition (4) is fulfilled. Here, we provide the general version of this result:

Proposition 2'. *[Risk Aversion and Fund Preference] Investor i with borrowing bound $\bar{l}$ prefers fund j over fund k, with $\beta_j > \beta_k$, if and only if $\gamma_i < \overline{\gamma_{jk}}$, with*

$$\overline{\gamma_{jk}} = \begin{cases} \frac{\beta_j \mu_M - \phi_j}{\beta_j^2 \sigma_M^2 \bar{l}}, & \textit{for } \beta_j\left((\beta_k - \beta_j)^2 \mu_M + 2\beta_j \phi_k\right) = \left(\beta_k^2 + \beta_j^2\right)\phi_j \\ 2\frac{\mu_M(\beta_j - \beta_k) - (\phi_j - \phi_k)}{(\beta_j^2 - \beta_k^2)\sigma_M^2 \bar{l}}, & \textit{for } \beta_j\left((\beta_k - \beta_j)^2 \mu_M + 2\beta_j \phi_k\right) < \left(\beta_k^2 + \beta_j^2\right)\phi_j \\ \frac{\beta_k \beta_j \mu_M - \sqrt{(\beta_j \phi_k - \beta_k \phi_j)(-2\beta_k \beta_j \mu_M + \beta_j \phi_k + \beta_k \phi_j)} - \beta_j \phi_k}{\beta_k^2 \beta_j \sigma_M^2 \bar{l}}, & \\ & \textit{for } \beta_j\left((\beta_k - \beta_j)^2 \mu_M + 2\beta_j \phi_k\right) > \left(\beta_k^2 + \beta_j^2\right)\phi_j. \end{cases} \tag{15}$$

Note that in the third case, the risk aversion "cutoff" value $\overline{\gamma_{jk}}$ depends non-linearly on the fund betas and fees, and a numerical solution of the model is required in this case.

Equilibrium solution for linear case In the second case of Proposition 2', where condition (4) is fulfilled, the first order conditions for the fund manager optimization problems (2) constitute a linear equation system $A\phi = b$, where ϕ is the vector of all fund fees, A is the tridiagonal matrix

$$A = \begin{pmatrix} \frac{2}{\beta_1^2 - \beta_2^2} & \frac{1}{\beta_2^2 - \beta_1^2} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{1}{\beta_2^2 - \beta_1^2} & \frac{2(\beta_1^2 - \beta_3^2)}{(\beta_3^2 - \beta_2^2)(\beta_2^2 - \beta_1^2)} & \frac{1}{\beta_3^2 - \beta_2^2} & \ddots & & & \vdots \\ 0 & \frac{1}{\beta_3^2 - \beta_2^2} & \frac{2(\beta_2^2 - \beta_4^2)}{(\beta_4^2 - \beta_3^2)(\beta_3^2 - \beta_2^2)} & \frac{1}{\beta_4^2 - \beta_3^2} & \ddots & & \vdots \\ \vdots & \ddots & \frac{1}{\beta_4^2 - \beta_3^2} & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \ddots & \ddots & \frac{1}{\beta_J^2 - \beta_{J-1}^2} \\ 0 & \cdots & \cdots & \cdots & 0 & \frac{1}{\beta_J^2 - \beta_{J-1}^2} & -\frac{2}{\beta_J^2 - \beta_{J-1}^2} \end{pmatrix}, \tag{16}$$

and

$$b = \begin{pmatrix} \overline{\Gamma}\sigma_M^2/2 - \mu_M \frac{1}{\beta_1 + \beta_2} \\ \mu_M \frac{\beta_3 - \beta_1}{(\beta_1 + \beta_2)(\beta_2 + \beta_3)} \\ \mu_M \frac{\beta_4 - \beta_2}{(\beta_2 + \beta_3)(\beta_3 + \beta_4)} \\ \vdots \\ \mu_M \frac{1}{\beta_{J-1} + \beta_J} - \underline{\Gamma}\sigma_M^2/2 \end{pmatrix}. \tag{17}$$

Clearly, the solution of the linear equation system can be obtained analytically for an arbitrary number of funds J.

Constant values in equilibrium solution For the case $J = 3$, with fund 1 being the market ETF, the solution is given by (9) for the case where all investors are strictly borrowing constrained, and by (15) for the more general case of relaxed borrowing constraints for some investors.

The related constants, which are all expressions of the vector of betas, are given by

$$\begin{aligned} A_1 &= (\beta_2 - \beta_M)(\beta_2 + 2\beta_3 - \beta_M), \\ A_2 &= (2\beta_3^2 - \beta_2^2 - \beta_2\beta_3 + (\beta_2 + \beta_3)\beta_M - 2\beta_M^2), \\ B_1 &= (\beta_2 + \beta_3)(\beta_2 - \beta_M)(\beta_2 + \beta_M), \\ B_2 &= (\beta_2 + \beta_3)(2\beta_3^2 - \beta_2^2 - \beta_M^2), \\ C &= \frac{\beta_3 - \beta_2}{4\beta_3^2 - \beta_2^2 - 3\beta_M^2}. \end{aligned} \tag{18}$$

Note that all constants are positive, which becomes obvious when using that all betas here are greater than 1 and ordered by magnitude.

Proof of Proposition 4 To prove the Proposition, let us first state the equilibrium solution for general ψ and $\bar{l}$, which is given by:

$$\begin{aligned} \phi_2 - \phi_M &= \frac{1}{C}(A_1\mu_M - \frac{\bar{l}}{2(1 + (\bar{l} - 1)\psi)}B_1\underline{\Gamma}\sigma_M^2), \\ \phi_3 - \phi_2 &= \frac{1}{C}(A_2\mu_M - \frac{\bar{l}}{2(1 + (\bar{l} - 1)\psi)}B_2\underline{\Gamma}\sigma_M^2). \end{aligned} \tag{19}$$

Note that for $\psi = 1$ or $\bar{l} = 1$, we are back to the solution stated in (9). Part (i) of the Proposition then follows for the sufficient condition $\underline{\Gamma}\sigma_M^2 < \mu_M/\beta_3$, as described in the main text. For (ii), observe that $\frac{\partial \frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}}{\partial \psi} < 0$ and $\frac{\partial \frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}}{\partial \bar{l}} > 0$, from which the result follows. Part (iii) is an immediate implication of part (i), as raw alphas $\alpha_i' = R_i - \beta R_M$ are zero in the model.

Model calibration We calibrate the model to standard values for the expected market return and the stock market volatility. The absolute risk aversion of the least risk-averse investors is set to 0.25, and to 2 for the most risk-averse investors. We consider two scenarios: One where all investors are strictly borrowing constrained with $l = 1$, and one where only a proportion of $\psi = 0.25$ is strictly borrowing constrained, while the remaining investor population faces a relaxed constraint with $\bar{l} = 2$. Finally, we consider one case with "few" funds, i.e., the market ETF plus two $\beta > 1$-funds and an arbitrary number of $\beta < 1$-funds, as well as one case with "many" funds, i.e., four $\beta > 1$-funds in addition to the market ETF and others. Table 1 summarizes these parameters and scenarios.