Paying for Beta: Leverage Demand and Asset Management Fees

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Abstract

We examine how investor demand for leverage shapes asset management fees. In our model, investors' leverage demand generates a cross-section of positive fees even if all managers produce zero risk-adjusted returns. We find support for the model's novel predictions in the sample of the U.S. equity mutual funds: (1) fees increase in fund market beta precisely for beta larger than one; (2) this relation becomes stronger when leverage constraints tighten; and (3) low net alphas are especially common among high-beta funds. These results suggest that asset managers can earn fees above their risk-adjusted returns for providing their investors with leverage.

Keywords: Leverage; Financial Intermediation; Mutual Funds
JEL Classifications: G11, G23, L11, L13

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1 Introduction

Many investors delegate portfolio decisions to professional money managers and pay fees for the asset management service. The extent of delegation and the fee revenues have grown significantly over the last four decades.[1] French (2008) reports that individual investor holdings of U.S. common equity declined from 47.9% in 1980 to only 21.5% in 2007, while open-end mutual fund holdings increased from 4.6% to 32.4% over the same period.[2] At the same time, investors sacrificed about 10% of their annual real return for asset management fees and transaction costs. The variation in fees represents a long-standing puzzle for financial economists since many funds charge fees which are significantly higher than their risk-adjusted returns.[3] In this paper, we develop and test a new theory of fee determination, suggesting that asset managers can charge fees for provision of leverage to investors who face borrowing constraints.

Our basic idea can be illustrated through the following example. Consider two investors with different risk profiles who need to choose an asset manager and can easily obtain leverage. The risk-seeking investor seeks an above-the-market return with a market beta of 1.5, while the risk-averse investor seeks a below-the-market return with a beta of 0.5. To obtain the desired return, the risk-seeking investor borrows 50% of her wealth and makes a leveraged investment in a market index fund. The risk-averse investor equally splits her holdings between the index fund and the risk-free asset. But if the risk-seeking investor cannot borrow, she has to find a manager who can deliver a leveraged portfolio with a beta of 1.5. This manager can charge an extra fee for providing leverage to the constrained risk-seeking investor, irrespective of the potential fees associated with the manager's risk-adjusted return.

[1] In 2018, only the U.S. equity mutual fund investors paid more than $50B in fees. This calculation is based on the Investment Company Institute 2019 report. The total mutual fund industry assets under management as of December 2018 amount to $17.7T, where equity funds represent 52% of assets. The value-weighted expense ratio for the equity funds equals 0.55%.

[2] Stambaugh (2014) extends the time series to 2012, providing consistent evidence on the long-term decline in direct equity ownership by individual investors.

[3] For early evidence on the underperformance of actively managed funds, see Jensen (1968), Ippolito (1989), and Gruber (1996). For the recent advancements, see, for example, Fama and French (2010), Del Guercio and Reuter (2014), Berk and van Binsbergen (2015), and Cremers, Fulkerson, and Riley (2019).

To sharpen this intuition, we present a new model in which investors delegate capital to asset managers. Asset managers differ in the amount of embedded leverage (Frazzini and Pedersen (2020)) they provide as measured by their market betas, and investors vary in their risk aversion. Managers compete on fees, and each investor needs to choose her preferred asset manager. If neither investors nor asset managers face leverage constraints, price competition drives fees towards zero across all managers.

Under leverage constraints, investors are willing to pay extra fees to high-beta asset managers because these managers provide returns that investors cannot obtain on their own. The willingness to pay for embedded leverage increases with the tightness of leverage constraints and declines with investor risk aversion. As a result, the model equilibrium features sorting of investors across managers such that risk-seeking investors invest with high-beta managers. Even if market index funds charge a very low fee, the managers with betas greater than one possess local market power over their constrained, risk-seeking investors. At this range of betas, fees progressively increase with beta across managers because managers with higher betas have more risk-seeking investor clienteles. At the same time, risk-averse investors do not require leverage since they look for below-the-market returns. These investors split their portfolios between the cheap market index fund and the risk-free asset. Consequently, fees of asset managers with betas smaller than one do not increase in beta. These results continue to hold even if asset managers themselves face non-trivial borrowing costs that reduce gross alphas of high-beta managers and increase gross alphas of low-beta managers as in Frazzini and Pedersen (2014) and Boguth and Simutin (2018).

Our model of leverage-based fees delivers three new testable hypotheses. First, the model predicts an asymmetric relation between market beta and fees across funds. In particular, fees increase in beta when beta is larger than one, but they are non-increasing in beta when beta is smaller than one. Second, the relation between beta and fees at the range of betas greater than one becomes stronger when leverage constraints tighten. Finally, the model predicts that funds' net alpha declines in beta and is particularly negative for beta greater than one. The effect of high fees on high-beta funds comes on top of the risk-return relation inherited from the asset market, through which portfolios of high-beta stocks may already have low gross alphas (Black, Jensen, and Scholes (1972), Frazzini and Pedersen (2014)). As a result, our theory suggests that net-of-fees underperformance is exacerbated for high-beta funds.

We examine the model's predictions in the sample of the U.S. domestic equity mutual funds. We first explore the asymmetric relationship between fund beta and fund fees for different ranges of betas as guided by the model. Implementing a variety of tests and controlling for the known determinants of fees, we confirm our first hypothesis: when beta is larger than one, fund fees increase with fund beta. When fund beta is below one, the relation between beta and fees becomes economically and statistically insignificant. The effect of beta on fees for betas above one is economically meaningful: when fund beta increases from 1 to 1.7, the top of our sample distribution, fund fees increase by 34 basis points, which is about a 22% increase relative to the median fee. This effect also stands as economically comparable to the effects of other determinants of fund fees. For example, according to our estimation, an increase of one standard deviation in log fund size is associated with a reduction of 21 basis points in fees, while an increase of one standard deviation in log fund age is associated with an increase of 9 basis points in fees. In terms of robustness, our findings are not confounded by differences in pricing policies across fund families, demand for style investing, differences in investors across fund distribution channels or a decline in fund offerings with fund beta.

We next explore our second hypothesis and examine whether the relation between beta and fees becomes stronger if leverage constraints are tight. We present two series of tests. The first group of tests is focused on the cross-sectional differences between institutional and retail investors. Our hypothesis is that the relation between beta and fees is stronger for share classes offered to retail investors, since they tend to face tighter leverage constraints.[4] We find that, for the same increase in beta, the increase in fees paid by retail investors is almost twice as large as for institutional investors. This result is consistent with the model's second prediction suggesting that constrained investors are willing to pay more for embedded leverage.

In our second series of tests, we examine the effects of time variation in leverage constraints on the cross-sectional relation between beta and fees. In particular, we compare funds that launched in periods of tight leverage constraints to funds launched in less constrained periods. We expect the relation between beta and fees to be stronger in the cross-section of funds launched in constrained periods. We use a number of measures associated with leverage constraints such as: (1) the betting-against-beta (BAB) factor from Frazzini and

[4] Frazzini and Pedersen (2014) show that individual investors are more likely to hold high-beta stocks, consistent with the intuition that they are more leverage-constrained.

Pedersen (2014); (2) the intermediary capital ratio (ICR) from He, Kelly, and Manela (2017); and (3) the leverage constraint tightness (LCT) measure from Boguth and Simutin (2018). We find that funds introduced in constrained periods charge two to four times more per unit of beta relative to funds introduced in less constrained periods. This is again in line with leverage-based fees: when investors are more constrained, they are willing to pay more to obtain leverage through their asset managers. Moreover, we provide additional evidence on fund flows that supports this interpretation since higher-beta funds exhibit higher net fund flows in more constrained periods.

We proceed to examine our third prediction and explore the implications of our model for fund net-of-fees performance. Using portfolio sorting, we first document that fund net alpha declines in fund market beta. In the sample of funds with betas greater than one, the difference in net alphas between the low-beta and the high-beta fund portfolios amounts to 60 basis points per year. We quantify the contribution of fees to this pattern by analyzing both gross and net alphas. Our analysis shows that high fees for high-beta funds and lower gross alphas contribute to the decline in net alpha on an approximately equal basis. These results suggest that demand for leverage plays an influential role in the low net-of-fees performance in the market of equity mutual funds.

Our empirical results consistently support the model, indicating that investors' leverage demand is an important driving factor of high fees for high-beta funds. As a complementary mechanism, fees could also be driven by higher costs incurred by asset managers for providing a high-beta fund. For example, such costs could result from a more frequent use of derivatives or short-selling, or from higher trading costs for high-beta stocks. We thoroughly explore this possibility by analyzing how funds' investment practices and trading costs interact with our results on the relation of embedded leverage and fees. To conduct this analysis, we supplement the main sample with data on fund investment practices collected from N-SAR filings, available in the EDGAR database on the U.S. Securities and Exchange Commission (SEC) website. These filings reveal whether a fund engages in borrowing, short-selling, or usage of various derivatives such as index options, futures, or stock options.

We first document that only 29% of funds with betas greater than one engage in any of the alternative investment practices associated with leverage. Moreover, high-beta funds are as likely to engage in these practices as the rest of the funds. This result holds no matter whether we examine all the practices combined or each practice separately. Within the sample of high-beta funds, fund beta does not depend on whether the fund borrows money, conducts short-selling, or uses derivatives. Taken together, our findings indicate that the vast majority of the high-beta funds obtain their embedded leverage by investing in high-beta stocks. Our conclusion remains unchanged if we measure the usage of alternative investment practices by comparison of the fund beta with the beta of its stock holdings, instead of relying on the N-SAR filings.

Finally, we analyze directly whether alternative investment practices and stock trading costs affect the relation between beta and fees. If certain investment practices for obtaining leverage result in higher fund management costs, these costs may influence the determination of fees, beyond our baseline demand-driven effect.[5] The direct stock trading costs can also affect our findings especially since most of the high-beta funds obtain their betas through holding high-beta stocks. We find that our results for high-beta funds remain quantitatively the same, no matter if these funds primarily invest in high-beta stocks or if they engage in borrowing, usage of derivatives, or short-selling. Moreover, the relation between beta and fees is similar across funds which face high and low stock trading costs. In sum, the combined evidence suggests that the relation between beta and fees does not depend on funds' investment practices or stock trading costs.

1.1 Contributions to the Literature

Our key contribution is to develop and test a new theory of leverage-based price competition in asset management. In doing so, we present a novel perspective on the underperformance of money managers complementing other explanations such as the presence of non-sophisticated investors (Gil-Bazo and Ruiz-Verdu (2008), Gârleanu and Pedersen (2018)), time variation in performance (Glode (2011)), or weak incentives to generate performance (Del Guercio and Reuter (2014)). Unlike these papers, we argue that fees are not paid solely for performance, and we do not require investors to be insensitive to either fees or performance. Our theory suggests that even if all managers generate zero alphas, some sufficiently sophisticated but

[5] We do not have any reason to assume a priori which investment practices are more expensive. For example, it is unclear whether providing leverage via derivatives is more expensive than providing leverage via high-beta stocks.

borrowing-constrained investors willingly pay above-zero fees to lever up their portfolios through asset managers.

Gennaioli, Shleifer, and Vishny (2014) and Gennaioli, Shleifer, and Vishny (2015) argue that managers can charge fees for providing access to financial markets even in the absence of superior performance. Our paper follows their general idea of delegation, but takes a different perspective. In their model, managers charge fees for providing access to any risky asset—even investing in the baseline market portfolio requires paying a fee. In the equilibrium, the fees are the same for all managers. In contrast, in our model investors are free to invest in the market portfolio for a zero fee but they are unable to lever it up. The equilibrium fees vary across managers due to the variation in embedded leverage and in the risk aversion of manager clienteles. As a result, our theory provides distinct predictions by means of the asymmetric relation between beta and fees in the cross-section of managers, which we confirm empirically.

Our paper fits the growing literature on the effects of leverage constraints in asset pricing. Building on the idea of Black (1972), Frazzini and Pedersen (2014) and Frazzini and Pedersen (2020) show that leverage constraints and demand matter for asset prices, and Boguth and Simutin (2018) argue that the time variation in the aggregate portfolio beta of mutual funds captures the variation in demand for leverage by asset managers themselves. Furthermore, leverage-constrained fund managers may prefer high-beta stocks due to benchmarking requirements (Christoffersen and Simutin (2017)). Lu and Qin (2019) use leveraged funds to estimate shadow costs of leverage while Dam, Davies, and Moon (2019) show that demand for leverage contributes to discounts on closed-end funds. Our novel contribution is to link the literature on leverage constraints to the literature on fee determination and fund net performance.

As such, this paper is also related to the literature on performance-based competition in delegated money management. In addition to the early work by Berk and Green (2004), recent theoretical research includes Cuoco and Kaniel (2011) and Kaniel and Kondor (2012). Christoffersen and Musto (2002), Khorana, Servaes, and Tufano (2008), Gil-Bazo and Ruiz-Verdú (2009), and Cooper, Halling, and Yang (2020) examine the determinants of mutual fund fees empirically.

The rest of the paper is organized as follows. In Section 2, we present our model and derive the key testable hypotheses. In Section 3, we describe our data and methodology. In Section 4, we empirically examine the model's testable hypotheses. We study the effects of fund investment practices and trading costs in Section 5. Concluding remarks are provided in Section 6.

2 Model

2.1 Setup

Our model has two dates and two types of agents: asset managers and investors. At time 0, asset managers set fees, and investors choose managers since we assume that investors do not manage portfolios of risky assets on their own.[6] At time 1, managers liquidate their portfolios and distribute net-of-fees assets to their investors. There is a set of $J+1$ asset managers who manage funds with different market betas $0 < \beta_0 < \beta_1 = \beta_M < \ldots < \beta_J$, where β_M stands for the asset manager who offers a market index fund. Asset managers charge fees ϕ_j per dollar invested. A fund with β_j has an expected before-fee excess return of $\mu_j = \beta_j \mu_M + (1-\beta_j)\xi$ and volatility $\sigma_j = \beta_j \sigma_M$ resulting from its portfolio holdings. Here, $\mu_M = E[R_M - R_f]$ and $\sigma_M^2 = Var[R_M]$ are the excess return and variance of the market portfolio, and R_f is the risk-free asset return. Our specification for fund returns nests the capital asset pricing model (CAPM) as a baseline setting which is obtained for $\xi = 0$. In addition, our model can incorporate the "betting against beta" (BAB) case where leverage constraints affect returns in the asset market. In this case, $\xi > 0$ represents the tightness of funding constraints for asset managers (Frazzini and Pedersen (2014), Boguth and Simutin (2018)).

[6]This assumption is typical for theories of delegated asset management. See, for example, Cuoco and Kaniel (2011), Gennaioli, Shleifer, and Vishny (2014), and Gennaioli, Shleifer, and Vishny (2015). Since we focus on how investors choose managers, we follow the literature on delegation and do not allow investors to trade in risky assets directly. This setting fits well the recent evidence on the prevalence of delegation and the significant decline in direct shareholdings by individual investors. Specifically, the individual investor holdings of U.S. common equity dropped from 47.9% in 1980 to around 20% in 2012 while the open-end mutual fund holdings increased from 4.6% to 32.4% over the same period (French (2008), Stambaugh (2014)). Similarly, we could have assumed that the investors face significant costs of selecting, trading, and rebalancing large diversified portfolios of many individual risky assets.

There is a unit measure of investors. Investors have constant absolute risk aversion (CARA) preferences and are heterogeneous with respect to their risk aversion γ_i, and each investor is endowed with a unit of wealth. Investors decide to invest a fraction ω_i of their wealth with one asset manager of their choice, while the remaining wealth is invested into the risk-free asset. Investors face heterogeneous borrowing constraints, $\omega_i \leq l$. In particular, a fraction ψ of the investors is strictly borrowing-constrained with $l = \underline{l} = 1$, while a fraction $1 - \psi$ faces a relaxed constraint with $l = \bar{l} > 1$.

We assume perfect supply-side competition for market index funds with $\beta_M = 1$, following the intuition that all market index funds are very similar, and entry barriers in this highly competitive segment are low.[7] As a result, the fee ϕ_M on the market index fund equals marginal production/management costs, which we set to zero for simplicity.[8] All asset managers with betas different from 1 offer differentiated products and are subject to monopolistic competition with the other funds.

Investors' Problem Each investor decides how much to invest into risky assets and also chooses an asset manager. Formally, investor i solves the problem

$$\max_{j,\omega_i^j} \; \omega_i^j(\mu_j - \phi_j) + R_f - \frac{\gamma_i}{2}{\omega_i^j}^2\sigma_j^2, \tag{1}$$

choosing an asset manager $j \in \{0, 1, \ldots, J\}$ with beta β_j and an investment weight $\omega_i^j \in [0, l]$ subject to the given borrowing constraint.

Asset Managers' Problem Each asset manager maximizes revenues that she generates from fees. Asset manager j solves the problem

$$\max_{\phi_j} \; \phi_j AUM_j(\phi_j), \tag{2}$$

[7]It would be an interesting avenue for future research to endogenize fund entry and exit in different segments. Our goal in this paper is to present a first model that connects investors' leverage demand with asset management fees in equilibrium.

[8]Some market index funds charge fees which are exactly zero, and many major index funds charge fees which are very close to zero. For example, the Fidelity ZERO Total Market Index Fund seeks to replicate returns of the entire U.S. equity market, while charging a zero fee. The Vanguard Total Stock Market Index Fund charges a fee of 4 basis points, and is also available as an ETF for 3 basis points. Incorporating a small non-zero fee for the market index fund does not affect the economic implications of our model.

where AUM_j are the assets under management that are allocated to j when the fee is set to ϕ_j. As asset managers operate under monopolistic competition, they take the investors' demand function as given when maximizing revenues.

2.2 Investor Choice and Fund Assets Under Management

We next examine the investors' investment choices, which ultimately determine the funds' assets under management. We assume that an asset manager j survives in equilibrium only if some investors prefer j over all other managers in the universe.

Investor Choice Without loss of generality, suppose that investor i decides to invest with asset manager j. Then the first order condition for the weight of the risky investment is

$$\widetilde{\omega}_i^j = \frac{\mu_j - \phi_j}{\gamma_i \sigma_j^2}, \tag{3}$$

and i chooses her investment to be ${\omega_i^j}^* = \min\{\widetilde{\omega}_i^j, l\}$ due to the borrowing constraint.

We describe the investor's choice between different asset managers and show first that investors do not invest with asset managers whose fees are too high, either in an absolute sense or relative to other managers. All proofs are provided in the appendix.

Proposition 1. *[Dominated Funds] Investors do not invest into funds j with*

1. $\phi_j \geq \mu_j$ or with
2. $\phi_j > \frac{\beta_j}{\beta_k}\phi_k + \xi(1 - \frac{\beta_j}{\beta_k})$ for a fund k with $\beta_j < \beta_k$.

In Proposition 1, we provide necessary conditions for asset managers to have positive assets under management and to survive in equilibrium. The first part states that no investor is willing to invest with a manager whose expected after-fee excess return $\mu_j - \phi_j$ is smaller or equal zero. In the second part, we lay out the basic logic for our main result. In particular, the fees of asset managers with smaller betas are bounded by the fees of higher-beta managers. For illustration, consider the case in which the CAPM holds in the asset market ($\xi = 0$). In this case, equilibrium fees must be non-decreasing in betas since investors can always synthesize a lower-beta fund by investing in a fund with higher beta and holding a cash position. This argument does not apply the other way round: investors cannot synthesize a high-beta fund by a leveraged investment in a lower-beta fund due to borrowing constraints. As a result, asset managers with low betas cannot charge higher fees than asset managers with higher betas.[9]

We next characterize the investment decision of an individual investor given her risk aversion γ_i. By comparing the levels of utility provided by two funds j and k, with $\beta_j > \beta_k$ and optimal investment weights $\omega_i^{j^*}$ and $\omega_i^{k^*}$, we show that investors prefer fund j over k if their risk aversion is below a certain threshold, which we denote by $\overline{\gamma_{jk}}$.[10] For notational ease, define $\widetilde{\mu_M} = \mu_M - \xi$.

Proposition 2. *[Risk Aversion and Fund Preference] Investor i with borrowing bound l prefers fund j over fund k, with $\beta_j > \beta_k$, if and only if $\gamma_i < \overline{\gamma_{jk}}$, with*

$$\overline{\gamma_{jk}} = 2\frac{\widetilde{\mu_M}(\beta_j - \beta_k) - (\phi_j - \phi_k)}{(\beta_j^2 - \beta_k^2)\sigma_M^2 l}. \quad (5)$$

We illustrate this result in Figure 1 and show the combinations of investor risk aversion γ_i and fee ϕ_j for which an asset manager j dominates the market index fund with $\beta_M = 1$ and $\phi_M = 0$. In both plots, the yellow area stands for the region in which the asset manager with $\beta = 1.3$ is preferred to the market index fund. Investors with very low risk aversion are willing to pay a lot for leverage and prefer the high-beta asset manager over the market index fund even if the asset manager charges a very high fee. The fee at which the high-beta fund j is preferred declines in investor risk aversion.

Comparing this to the blue area—the region in which an asset manager with lower beta ($\beta = 1.1$) is preferred to the market index fund—highlights the effect on fees across asset managers with different betas. The yellow area overlays the blue area: the manager with $\beta = 1.3$ can set higher fees and still be strictly preferred by some investors over the market

[9]If ξ is substantially greater than zero, the restriction on fees through Condition 2 of Proposition 1 is somewhat relaxed, but we show that in the model equilibrium fees increase in beta particularly for $\beta > 1$.

[10]For ease of exposition, we present our result for the case that the condition

$$\beta_j(\beta_k - \beta_j)^2(\mu_M - \xi) < \phi_j\left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2\phi_k + \xi\left(\beta_j^2 - \beta_k^2\right) \quad (4)$$

holds, for which the "cutoff" $\overline{\gamma_{jk}}$ is linear in fees. We discuss the general case in the appendix.

index fund. As risk aversion declines, the investor is willing to pay significantly more to the high-beta asset manager even if the low-beta manager is available.

Finally, we graphically illustrate the role of the tightness of leverage constraints. The left plot describes the choice of an investor who faces strict constraints ($l = 1$), while the right plot presents the more relaxed case ($l > 1$). In the strict case, investors cannot obtain leverage by any means. As a result, even investors with moderate risk aversion prefer high-beta asset managers over the market index fund if the fee is not too extreme.

We extend this logic further and show that in equilibrium, investors sort across managers depending on their betas, and the corresponding investor clienteles are formed based on risk aversion.

Proposition 3. *[Investor Clienteles] For all funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$, it must be that $\overline{\gamma_{j_2 j_1}} < \overline{\gamma_{j_1 k}}$ and $\overline{\gamma_{j_2 k}} < \overline{\gamma_{j_1 k}}$ in equilibrium. Asset managers with higher betas are chosen by investors with lower risk aversion.*

We illustrate this result in Figure 2. In the equilibrium, asset managers with different betas offer their services to different types of investors. In particular, investors with the lowest risk aversion choose the asset manager with the highest beta, up to a certain cutoff point, after which the second-least risk-averse clientele chooses the fund with the second-highest beta, and so on.

Using the results from Propositions 2 and 3, we can compute the assets under management (AUM) of fund j, dependent on the fee ϕ_j. In particular, the AUM are given by

$$AUM_j(\phi_j) = \int_{\overline{\gamma}_{j+1,j}}^{\overline{\gamma}_{j,j-1}} f(\gamma_i) d\gamma_i, \tag{6}$$

where the integration bounds are defined in line with Proposition 2, and $f(.)$ is the probability density for the risk aversion in the investor population. We utilize the fact that asset manager j attracts investors whose risk aversion is below the threshold $\overline{\gamma}_{j,j-1}$ at which the manager with the next-lower beta is dominated, but larger than the value $\overline{\gamma}_{j+1,j}$ at which manager j is dominated by the manager with the next-higher beta.

2.3 Equilibrium

An equilibrium is a combination of fees $\phi_0, \phi_1, \ldots, \phi_J$ for the asset managers such that, for optimal investor choices, fee revenues are maximized for all asset managers according to (2). To solve for the model equilibrium explicitly, we need to make an assumption on the probability distribution of γ_i. We assume that γ_i is equally distributed on $[\underline{\Gamma}, \overline{\Gamma}]$. The model can be solved analytically for the case considered in Proposition 2, for which condition (4) is fulfilled. For other cases, we can efficiently compute the equilibrium numerically. In the analytical case, the first order conditions obtained from the fund manager optimization problems (2) constitute a linear equation system $A\phi = b$, where ϕ is the vector of all fund fees, and A is a tridiagonal matrix.

Let us explicitly demonstrate and explore the equilibrium solution for the case of four funds with betas $0 < \beta_0 < \beta_1 = \beta_M = 1 < \beta_2 < \beta_3$, starting with $\xi = 0$ and $\psi = 1$ for ease of exposition.[11] Since there is perfect supply-side competition for market index funds with $\beta_1 = \beta_M = 1$, the index fund fee ϕ_M equals the marginal management cost which is zero. Proposition 1 then implies that for $\xi = 0$ the fee ϕ_0 for the asset manager with $\beta_0 < 1$ is zero too; otherwise, it would always be optimal for investors to invest in the market index fund and cash in order to replicate the fund with β_0 at zero fees. The same argument holds for potential additional asset managers with beta smaller than one, such that fees become flat in betas for $\beta < 1$.

We next solve for the fees ϕ_3 and ϕ_2 of the funds with $\beta_3 > \beta_2 > 1$, which are set by their managers under monopolistic competition. The revenue maximization problems (2), in which we insert the assets under management computed according to (6) with uniformly distributed γ_i, are obtained as

$$\begin{aligned}
&\max_{\phi_2}\ \phi_2 \cdot \frac{1}{\overline{\Gamma} - \underline{\Gamma}} \left(2\frac{\widetilde{\mu_M}(\beta_2 - \beta_M) - (\phi_2 - \phi_M)}{(\beta_2^2 - \beta_M^2)\sigma_M^2} - 2\frac{\widetilde{\mu_M}(\beta_3 - \beta_2) - (\phi_3 - \phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} \right), \\
&\max_{\phi_3}\ \phi_3 \cdot \frac{1}{\overline{\Gamma} - \underline{\Gamma}} \left(2\frac{\widetilde{\mu_M}(\beta_3 - \beta_2) - (\phi_3 - \phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} - \underline{\Gamma} \right).
\end{aligned} \tag{7}$$

Given the fees, all investors with low enough risk aversion, down to the lowest risk aversion $\underline{\Gamma}$, prefer the β_3 manager over the β_2 manager. These investors invest with the β_3 manager

[11] The linear equation system for an arbitrary number of $J + 1$ funds is provided in the appendix.

since there are no managers with higher beta. Another group of investors invests with the β_2 manager. These investors are more risk-averse relative to the first group and prefer the β_2 manager over the β_3 manager. At the same time, these investors still have low enough risk aversion such that they do not invest in the market index fund. The rest of the investors chooses the market index fund. Given the investor demand, the fund managers maximize revenues by setting the appropriate fees.

Taking the derivatives of the fund managers' objective functions by ϕ_2 and ϕ_3, respectively, and setting them to zero, yields the corresponding first order conditions

$$
\begin{aligned}
2\frac{\widetilde{\mu_M}(\beta_2-\beta_M)-(2\phi_2-\phi_M)}{(\beta_2^2-\beta_M^2)\sigma_M^2} - 2\frac{\widetilde{\mu_M}(\beta_3-\beta_2)-(\phi_3-2\phi_2)}{(\beta_3^2-\beta_2^2)\sigma_M^2} &= 0, \\
2\frac{\widetilde{\mu_M}(\beta_3-\beta_2)-(\phi_3-2\phi_2)}{(\beta_3^2-\beta_2^2)\sigma_M^2} - \underline{\Gamma} &= 0.
\end{aligned} \tag{8}
$$

Since $\phi_M = 0$, we can solve the given system of two equations for the two fee variables, ϕ_2 and ϕ_3. The solution can be written as

$$
\begin{aligned}
\phi_2 - \phi_M &= \frac{1}{C}(A_1\widetilde{\mu_M} - \frac{1}{2}B_1\underline{\Gamma}\sigma_M^2), \\
\phi_3 - \phi_2 &= \frac{1}{C}(A_2\widetilde{\mu_M} - \frac{1}{2}B_2\underline{\Gamma}\sigma_M^2),
\end{aligned} \tag{9}
$$

where the constants $A_1, A_2, B_1, B_2, C > 0$ result from the vector of betas. In the appendix, we define these constants and also show that the solution for heterogeneous borrowing constraints (i.e., $0 < \psi < 1$) has the same form and is obtained by replacing $\frac{1}{2}$ with $\frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}$.

We next explore the model equilibrium and derive the relation between the amount of leverage that the manager provides (as captured by her market beta) and fees. In (9), for both expressions the $\widetilde{\mu_M}$ term is greater than the negative σ_M^2 term for all relevant combinations of the given parameters.[12] It follows that for beta greater than one, fees increase in beta. We summarize this result and its implications in the following proposition.

[12]A sufficient condition is $\underline{\Gamma}\sigma_M^2 < \widetilde{\mu_M}/\beta_3$. In our benchmark calibration (see Table 1), it is $\underline{\Gamma}\sigma_M^2 = 0.01$ and $\widetilde{\mu_M}/\beta_3 = 0.0206$ in the betting-against-beta scenario, comfortably fulfilling this condition. In the CAPM case, $\widetilde{\mu_M}/\beta_3 = 0.0294$.

Proposition 4. *[Paying for Beta] Suppose* $0 < \beta_0 < \beta_1 = \beta_M = 1 < \beta_2 < \beta_3$*. In this case:*

(i) $\phi_2 - \phi_M > 0$ *and* $\phi_3 - \phi_2 > 0$*. Managers with higher beta earn higher fees, if beta is greater than one.*

(ii) $\frac{\partial(\phi_2-\phi_M)}{\partial\psi} > 0$, $\frac{\partial(\phi_3-\phi_2)}{\partial\psi} > 0$, $\frac{\partial(\phi_2-\phi_M)}{\partial\bar{l}} < 0$, $\frac{\partial(\phi_3-\phi_2)}{\partial\bar{l}} < 0$*. The increase of fees in beta for beta greater than one becomes steeper when investors face tighter borrowing constraints, i.e., when the fraction* ψ *of strictly constrained investors increases, or when* $\bar{l}$*, the borrowing bound of less constrained investors, decreases.*

(iii) if manager net performance relative to the CAPM is defined as $\alpha_j = \mu_j - \beta_j\mu_M - \phi_j$*, then* $\alpha_2 < \alpha_M$ *and* $\alpha_3 < \alpha_2$*. Managers' net performance is strictly decreasing in beta for managers with betas greater than one.*

The intuition behind Proposition 4 is straightforward, and can been seen in Figure 3 which depicts the relation between betas and fees. We calibrate the model for multiple scenarios using the parameter values from Table 1. The blue line refers to the baseline relationship. Investors with low enough risk aversion choose the asset manager with $\beta_3 = 1.7$. Since these investors have higher willingness to pay for embedded leverage, they pay the highest fee in equilibrium. The next group of investors choose the asset manager with $\beta_2 = 1.3$ and pay a lower fee. More risk-averse investors invest in the market index fund with $\beta_M = 1$. The most risk-averse investors are indifferent between the asset manager with $\beta_0 = 0.3$ or investing in the market index fund plus cash, such that fees for beta smaller than one are bounded by the market index fund.

The yellow line refers to the setting with tighter leverage constraints. In this case, the willingness to pay for embedded leverage increases for all the investors, and the asset managers with betas above one can charge even higher fees for the same beta. As a result, the scenario of tighter borrowing constraints features an increased slope of the beta-fee relation.

The green line refers to the setting with a larger number of funds, for which we solve the model numerically. The relation between beta and fees remains the same.

Finally, the orange line presents a scenario with a considerable BAB effect in the asset market. In this case, fees of low-beta funds may decline in beta since fund gross alpha declines in beta. The potential decline for beta smaller than one is very modest for sensible calibrations. Intuitively, the decline becomes more pronounced when the BAB effect is stronger ($\xi \gg 0$), but in such a scenario the fund investors' demand for low-beta funds will likely also be lower (i.e., $\overline{\Gamma}$ will be lower), counteracting the effect. At the same time, the relation between beta and fees remains positive for funds with beta greater than one, while flattening slightly.

Our results have a direct implication for the fund net performance as measured by the CAPM net-of-fee alpha. In our baseline case, gross alphas for all funds equal zero by definition since the CAPM holds. As a result, each fund's net alpha equals minus the fee. Since fund fees are increasing in fund beta when beta is larger than one, net alpha must be decreasing in beta. Intuitively, investors pay for provision of leverage, and the value of this service is not captured by the manager's net alpha. If we add the BAB effect in the asset market in the form of $\xi > 0$, net alpha will further decrease in beta due to the additional negative effect of high beta on gross alpha.

2.4 Testable Hypotheses

In our empirical work, we examine three specific hypotheses that are implied by Proposition 4. We first formulate our hypothesis regarding the baseline asymmetric relation between beta and fees across funds.

Hypothesis 1. *After controlling for the known determinants of fees, fees increase with beta for funds with beta larger than one, and fees are non-increasing in beta for funds with beta smaller than one.*

Hypothesis 1 follows directly from Proposition 4(i). Since our theory focuses on the effects of embedded leverage on fees, it is complementary to the effects of other known determinants of fees such as fund gross performance, its size, age, and fund family pricing policies (Gil-Bazo and Ruiz-Verdú (2009), Cooper, Halling, and Yang (2020)). Consequently, in our empirical work we have to include a proper set of control variables to test whether the effect of beta is unique and is not being subsumed by other variables known to explain fees.

Since our model suggests that leverage constraints drive the relation between beta and fees, it is natural to explore how the relation varies with the tightness of leverage constraints. This motivates the second hypothesis.

Hypothesis 2. *The relation between beta and fees for funds with beta larger than one is stronger*

(i) for funds held by retail investors than for funds held by institutional investors,

(ii) for funds which are introduced to the market during periods of tight borrowing constraints relative to funds introduced in less constrained periods.

Hypothesis 2 follows from Proposition 4(ii). The relation between beta and fees becomes stronger when either the fraction of strictly constrained investors increases, or when less constrained investors face a lower borrowing limit. In line with Frazzini and Pedersen (2014), we assume that retail investors face more severe leverage constraints relative to institutional investors, and we utilize this difference in the cross-section of investor types in the first part of Hypothesis 2. In terms of the theory, we can think about this hypothesis in two ways. First, retail investors as a group can have a higher fraction of individuals who are severely constrained. Second, the borrowing limit of less constrained retail investors can be lower than the borrowing limit of less constrained institutional investors.

The second part of Hypothesis 2 is also implied by Proposition 4(ii). The tightness of leverage constraints varies not only in the cross-section of investors but also over time (Frazzini and Pedersen (2014), He, Kelly, and Manela (2017), Boguth and Simutin (2018)). If either the fraction of constrained investors or the borrowing limit varies over time, then the strength of the relation between beta and fees is expected to vary as well. While our model considers a static setting which does not directly generate predictions regarding time variation in beta and fees within a given fund, it still has an implication for the funds that are launched in different time periods. In particular, the funds introduced to the market in times of tight leverage constraints should have a stronger relation between beta and fees relative to the funds introduced in times of weak leverage constraints. In our empirical work, we focus on specific time-varying measures of leverage constraints to test this hypothesis.

Since funds with higher betas charge higher fees, our model has a direct implication for fund net performance. This implication is derived in our third hypothesis.

Hypothesis 3. *When betas are larger than one, fund net CAPM alpha declines in beta faster than gross CAPM alpha.*

Hypothesis 3 follows from Proposition 4(iii). As fees increase in fund beta for betas greater than one, our theory suggests that fund net alphas should decline with beta due to the effect of fees. Importantly, we do not argue that fees are the only driving factor of the relation between beta and net alpha. For example, a relatively flat security market line in the asset market (see Black, Jensen, and Scholes (1972), Frazzini and Pedersen (2014)) implies that stocks with high beta have low alpha. As a result, funds with higher beta can have a lower gross alpha which results in a lower net alpha. However, our model suggests that fees can further reduce net alphas of high-beta funds beyond what is already implied by their portfolio holdings. As a result, when beta increases, fees progressively increase the gap between net and gross performance. Consequently, if we sort funds into portfolios with respect to their betas, we expect net alphas to decline in beta faster than gross alphas.

3 Data and Methodology

3.1 Data and Variables

In this section, we describe our main dataset and the construction of its key variables. We obtain our data from the CRSP U.S. Mutual Fund Database for the period from January 1991 to December 2016. Our sample starts in 1991 because monthly reporting of fees, total net assets, and investment objectives becomes consistent and precise after 1990 (see also Gil-Bazo and Ruiz-Verdú (2009)). We start with the initial sample of all open-end mutual funds and keep only domestic equity funds using the information on fund investment objectives. We identify passive funds and exchange-traded funds (ETF) based on the CRSP definitions. To obtain a proper estimate of fund ownership costs to investors, we combine the information on fund annual expense ratios and loads. We follow Sirri and Tufano (1998) and Gil-Bazo and Ruiz-Verdú (2009) assuming an average fund share holding period of seven years. As a result, we define the mutual fund total annual fee as the sum of the fund's annual expense ratio and one-seventh of the sum of the front load and the back load.

We use three different datasets in our tests: the fund share class dataset, the fund-level dataset, and the fund launch dataset. We obtain the fund share class dataset directly from the CRSP database. To construct the fund-level dataset, we calculate the averages of the

CRSP variables across the share classes within a fund for each month, weighted by the share class total net assets in that month. To obtain the fund launch dataset, we define the month of a share class's first appearance in the CRSP database as the month of its launch, and collect the fund share class data only for this month.

3.2 Estimation of Market Beta and Fund Performance

We estimate the market model with a rolling window to evaluate a fund's market beta and its performance relative to the market portfolio in each month. Specifically, we estimate the following time-series regression for each fund:

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{Mt} - R_{ft}) + e_{it}. \tag{10}$$

In this regression, R_{it} is the return on fund i for month t, R_{ft} is the 1-month U.S. Treasury bill rate, R_{Mt} is the market return obtained from Kenneth French's website, and α_i is the average return unexplained by the market model that we annualize and further refer to as an estimate of fund CAPM alpha. We use two variants of this model following Fama and French (2010). The first variant uses fund net returns to estimate fund net alpha, and the second variant uses fund gross returns to estimate fund gross alpha. We define the monthly fund gross return as a sum of the monthly fund net return and one-twelfth of the annual fund fee. We refer to the estimate of β_i as the fund market beta. We present our results based on the estimates of fund betas derived from fund gross returns, but they remain virtually unchanged if we use betas derived from fund net returns.

To estimate the models of fund performance, we follow Gil-Bazo and Ruiz-Verdú (2009) and require the fund to have at least 48 months of performance data available in the last 5 years, and we use 5-year rolling regressions to obtain estimates for each month. As a result, our estimates of fund performance and fund market beta become available after January 1995. For the fund launch sample, we estimate the models for the first 48 months of fund operation. We drop funds with extremely high fees in the sample (those above 99.9% of the sample distribution) and focus on betas in the middle 95% (i.e., 2.5%–97.5%) of the sample distribution.[13]

We present the distribution of the number of funds across market betas in Figure 4. The distribution is almost symmetric with many fund offerings concentrated around betas in the range of 0.9–1.1. As beta moves away from one, the number of fund offerings declines. Most of the U.S. equity mutual funds have market betas in the range of 0.2–1.7.

3.3 Summary Statistics

We present the summary statistics for our variables in Table 2. The information at the fund share class level is shown in Panel A. The average annual fee over the sample period is 1.57%, and its standard deviation is 0.75%. The standard deviation of fees within a given fund over time equals only 0.03%, indicating that almost all the variation in fees is driven by the differences across funds rather than the time variation within funds. The distribution of fees is relatively symmetric across funds as the median fee equals 1.52%. The funds at the top 5% of the fee distribution charge a 2.76% fee while the funds at the bottom 5% of the distribution charge only 0.37%.

The average fund market beta equals 1, with a standard deviation of 0.21, and a within-fund standard deviation of 0.04. Similar to fund fees, there is significantly more variation in market beta across funds than within funds. The average gross CAPM alpha equals 1.33% (t-stat = 10.6, with standard errors clustered by month), while the average net alpha equals -0.26% (t-stat = -2.28), suggesting that the returns to investors turn negative on average due to the effect of fees. Passive funds represent 7% of the share-class-months in our sample and ETFs represent 2%.[14]

We report the summary statistics at the fund level in Panel B. The average fee equals 1.38%, and it is lower relative to the fee from the fund share class data since the high-fee

[13]Our results are robust under different data cleaning criteria that drop more (e.g., those above 99%) or fewer (e.g., those above 99.99%) extremely high fees, or that focus on a narrower (e.g., the middle 90%) or wider (e.g., the middle 98%) range of betas.

[14]The definitions of a passive fund and an ETF do not necessarily overlap. A fund can be defined both as a passive fund and an ETF as in the example of any index-linked ETF. Index mutual funds meet the definition of passive funds but not of ETFs. In addition, some ETFs do not follow any index and are therefore considered to be actively managed.

share classes tend to have less assets under management. The distributions of beta and alpha are very similar to those from the share-class-level data. The average gross alpha equals 1.45% (t-stat = 10.7), while the average net alpha equals only 0.04%, statistically indistinguishable from zero (t-stat = 0.31). This is again in line with larger funds charging lower fees. The distributions of fees, betas, and alphas are roughly the same in the sample of fund launches (Panel C).

We next examine what drives variation in fees. The preliminary comparison of the standard deviation across funds to the standard deviation within funds has already revealed that fees and beta vary substantially more across funds than within funds. A more formal analysis of the relative importance of cross-sectional and time-series variations can guide us to properly design our empirical tests.

We report the R-squared from the regressions of variables on fund share class fixed effects and fund fixed effects in the last columns of Panels A and B, respectively. The time-invariant characteristics drive 96% of the variation in fees in the share class sample, and 90% of variation in fees in the fund-level sample. Furthermore, the time-invariant characteristics are responsible for 70% of the variation in betas in the share class sample and 66% of the variation in betas in the fund-level sample. These statistics suggest that almost all the variation in fees and most of the variation in betas is driven by differences across funds rather than within funds.

These data characteristics have two key implications for our analysis. First, we design our main tests based on the variation in fees and beta across managers, rather than on the variation within managers. Second, the evidence on the small time-series variation in our key variables fits our model which does not include dynamic time-series effects by its design.

4 Hypothesis Testing

4.1 Asymmetric Relation between Market Beta and Fund Fees

We examine the three testable hypotheses derived from our model. We start by testing Hypothesis 1 and examine the baseline relation between fees and embedded leverage as measured by fund market beta. Our baseline econometric specification is a panel regression of the form:

$$Fee_{ift} = \gamma_f + \gamma_t + \lambda Beta_{ift} + \rho X_{ift} + e_{ift}. \tag{11}$$

In this regression, Fee_{ift} is the fee for fund i in fund family f in month t, $Beta_{ift}$ is the fund market beta, γ_f is a fund family fixed effect, γ_t is a month fixed effect and X_{ift} is a set of fund-level time-varying control variables such as a fund's CAPM alpha, the logarithm of fund age in months, the logarithm of fund total net assets, an indicator variable that equals one if a fund is passively managed, and an indicator variable that equals one if a fund is an ETF. Standard errors are double-clustered by fund family and month. We use fund-months as a unit of observation in the fund-level tests and fund-share-class-months as a unit of observation in the fund share class tests. We include fund family fixed effects in our specifications to control for unobserved family-specific determinants of fees such as family pricing policies.

4.1.1 Main Tests

We first examine the relation between beta and fees non-parametrically. We present the binscatter plot of residual fees against market betas separately for funds with betas larger than one and smaller than one in Figure 5. The residual fee is estimated in two steps. First, we regress the fee on all the control variables and fixed effects as specified in (11), and then we calculate the residual fee as the original fee minus the predicted value from the estimation in the first step. The results in Figure 5 are consistent with the model's central predictions: (1) fees increase with market beta when beta is larger than one; and (2) fees are non-increasing in beta when beta is smaller than one.

We present the formal regression results in Table 3. The estimates from the share-class-level regressions in the first four columns confirm the graphical evidence from Figure 5. The results in columns (1) and (2) show that fees increase in beta when beta is larger than one. The coefficients on beta are statistically significant at the 1% level. The relation between betas and fees for betas above one is economically meaningful: when fund beta increases from 1 to 1.7, the top of our trimmed sample distribution, fund fees increase by

34 (0.48×0.7) basis points, which is about a 22% increase relative to the median fee. This relation also stands as economically significant relative to the effects of other determinants of fund fees. For example, an increase of one standard deviation in log fund size is associated with a reduction of 21 (0.09×2.34) basis points in fees, while an increase of one standard deviation in log fund age is associated with an increase of 9 (0.21×0.44) basis points in fees. In line with the theory, the estimate of the coefficient on beta for funds with betas below one is economically negligible and statistically indistinguishable from zero once we control for fund performance (columns (3) and (4)). The results also show that funds with higher CAPM alphas and active funds have higher fees.[15]

We next present the results for the fund-level data in columns (5)–(8). Overall, the estimates are very similar to those obtained through the share-class-level sample. The results in columns (5) and (6) show that for betas larger than one, the coefficients on beta are again large and significant. These coefficients exhibit a similar economic magnitude relative to the coefficients from the share-class-level regressions: when fund beta increases from 1 to 1.7, fund fees increase by 25 (0.35×0.7) basis points, which is about 19% of the median level. The estimate of the coefficient on beta for betas smaller than one is again very small and statistically indistinguishable from zero (columns (7) and (8)).

In sum, the combined evidence consistently supports Hypothesis 1. If borrowing-constrained investors pay fees for leverage, fees are expected to increase in beta only for funds with betas larger than one.[16]

4.1.2 Robustness to Fund Offerings across Betas

We discuss a number of robustness checks for our first main results. We first examine the robustness of these findings to the variation in the number of fund offerings with beta, as documented in Figure 4. Our concern is that fees may increase with beta due to the decline in

[15]Passive funds may exhibit non-zero alphas relative to the common benchmarks such as the CRSP value-weighted market portfolio (Cremers, Petajisto, and Zitzewitz (2012)). The results remain unchanged when we remove the passive funds from the tests that include fund performance as a control variable.

[16]Note that the asymmetry of the beta-fee relation, in line with our theory, sets a high hurdle for potential alternative explanations. For example, a behavioral explanation may suggest that investors are simply "naive", do not risk-adjust returns, and perceive higher total returns of high-beta funds as "fund performance". As a result, the investors may be willing to pay higher fees for higher beta. However, this explanation is difficult to reconcile with fees being flat in beta when beta is less than one.

the number of alternative choices, and not due to the effects of leverage demand. To address this concern, we construct two measures to capture the intensity of fund offerings within different ranges of betas. The first measure counts the overall number of funds for each 0.1-wide beta bin in a specific month (e.g., funds with betas between 0.8-0.9 are assigned to one bin, and funds with betas between 1.1-1.2 are assigned to another bin, etc.). The second measure computes the Herfindahl-Hirschman Index (HHI) for each beta bin in a specific month, where a fund's market share is defined as the fund's AUM divided by the AUM of all the funds in the same beta bin. We use the value of the respective intensity measure for all funds in the corresponding beta bin.

We estimate Equation (11) including the intensity measures in our specifications and report the results in Panel A of Table 4. For brevity, we only present the estimated coefficients on beta while the detailed results are reported in Table A1 in the appendix. Our main results remain unchanged, and the estimates of the coefficients on beta are quantitatively and qualitatively similar to the estimates from Table 3. The results are robust for both the fund share class sample (columns (1) and (2)) and the fund-level sample (columns (3) and (4)).

4.1.3 Robustness to Differences in Investors across Distribution Channels

We next explore whether the effects of beta on fees vary across distribution channels. Since the funds sold to investors via brokers have higher fees and higher beta relative to direct-sold funds (Del Guercio and Reuter (2014)), our results could be confounded by the differences in clienteles across these channels. To mitigate this concern, we examine the relation between beta and fees separately for direct-sold and for broker-sold funds. We follow Sun (2020) and consider a fund share class to be direct-sold if it charges no front or back load, and has an annual distribution fee ("12b-1 fee") of no more than 25 basis points; otherwise, a fund share class is considered as broker-sold.

We report the estimated coefficients on beta in Panel B of Table 4. The detailed results are reported in Table A2 in the appendix. The effect of beta on fees is quantitatively similar and statistically significant across the channels, suggesting that our results are robust to the differences in clienteles between direct-sold and broker-sold funds.

4.1.4 Robustness to Demand for Style Investing

Finally, we examine the effects of fund styles on our main results. Since investors seek for exposure to different types of stocks, fund fees may vary across styles (Gil-Bazo and Ruiz-Verdú (2009)). If funds in investment categories (styles) with high-beta stocks have higher fees, the relation between beta and fees may reflect the demand for style investing rather than the demand for leverage. To account for this, we add fund style fixed effects to our main specifications. We define fund styles using the Lipper classification of the U.S. equity funds, which constitutes the basis for CRSP fund style classifications.

We present the estimated coefficients on beta in Panel C of Table 4 while the detailed results are reported in Table A3 in the appendix. Accounting for style investing leads to more moderate estimated effects, but the coefficients remain statistically significant and large for the funds with betas greater than one relative to the funds with betas less than one. While the effect of beta holds even within styles, an alternative view is that the demand for a certain style may actually be caused by an underlying demand for leverage. In light of this view, obtaining smaller effects after controlling for fund style is not surprising and consistent with our other results.

4.2 Heterogeneity in Borrowing Constraints

4.2.1 Comparison of Retail and Institutional Investors

We proceed to examine Hypothesis 2 and explore variation in the tightness of borrowing constraints across investor types. We expect the relation between beta and fees for betas larger than one to be stronger among retail investors relative to institutional investors, since retail investors are more likely to face borrowing constraints (Frazzini and Pedersen (2014)). We test the prediction by introducing two indicator variables: a variable that equals one if a share class is offered to retail investors, and a variable that equals one if a share class is offered to institutional investors.[17] We add these two variables to our main specifications,

[17] Almost all share classes are offered either only to retail investors or only to institutional investors, and we remove the very few exceptions from our sample that are indicated to be offered to both investor types. Therefore, the institutional indicator is effectively one minus the retail indicator.

interacting them with market beta to evaluate the relation between beta and fees for different investor clienteles.

We present the results in Table 5. In the share class sample, the coefficient on the interaction between market beta and the indicator for the retail share class equals 0.44, while the coefficient on the interaction between market beta and the indicator for the institutional share class equals 0.30 (column (1)). This result indicates that the effect of beta on fees is almost 50% larger for retail share classes. The estimated coefficients remain virtually unchanged when we control for fund performance (column (2)). We formally test the significance of the difference between the coefficients, finding that this difference statistically significant at the 5% level, as reported in Table 5.

We next examine the robustness of our results in the sample of funds at launch, when the share class was first offered to the investors. Overall, we obtain similar findings, reported in columns (3) and (4). The coefficient on the interaction between market beta and the indicator for the retail share class equals 0.43, while the coefficient on the interaction between market beta and the indicator for the institutional share class equals 0.22. This result implies that when a share class is offered to retail investors, the fund family charges them almost twice as much for the same increase in beta relative to institutional investors. The difference between the coefficients for the fund launch sample is statistically significant with a p-value of 2%.

In sum, the comparison of retail and institutional share classes supports Hypothesis 2. More borrowing-constrained retail investors pay more for beta relative to less borrowing-constrained institutional investors.

4.2.2 Time Variation in Tightness of Borrowing Constraints

We next explore the effects of time variation in borrowing constraints. Hypothesis 2 suggests that the relation between beta and fees for betas larger than one is more pronounced in times when it is more difficult to borrow capital. We use three measures of borrowing constraint tightness: the betting-against-beta (BAB) factor from Frazzini and Pedersen (2014), the intermediary capital ratio (ICR) from He, Kelly, and Manela (2017), as well as the leverage constraint tightness (LCT) measure from Boguth and Simutin (2018). We use monthly variation in each measure and define periods when a measure takes on extreme values as constrained periods, separately for each measure. Low values of the BAB and the ICR measures as well as high values of the LCT measure indicate tighter borrowing constraints. Consequently, we define periods with the BAB or ICR measure in the first quartile of its time distribution or periods with the LCT measure in the fourth quartile as constrained periods. Accordingly, a time period is defined as unconstrained if the measure's value belongs to the opposite extreme quartile of its time distribution: the fourth quartile for the BAB and ICR measures, and the first quartile for the LCT measure.

We introduce two indicator variables separately for each measure: a variable that equals one if a period is defined as constrained, and a similar variable for unconstrained periods. We add these variables to our main specifications and interact them with market beta to evaluate the effects of time variation in borrowing constraints on the relation between beta and fees. Given the absence of the time-series variation in fees within funds, we examine the effects within the sample of funds at launch. These tests are also in line with our theory that focuses on the cross-sectional differences between asset managers. In particular, we test whether the funds introduced in constrained periods earn higher fees per unit of beta during this period relative to the funds introduced in unconstrained periods.

We present the results in Table 6, starting with the BAB factor as a measure of borrowing constraint tightness (columns (1) and (2)). The coefficient on the interaction between market beta and the indicator for constrained periods equals 0.54, statistically significant at the 1% level. At the same time, the coefficient on the interaction between market beta and the indicator for unconstrained periods equals 0.12, statistically indistinguishable from zero. The p-value of the test for the difference between the coefficients equals 3%. These results suggest that funds introduced in constrained periods, as measured by the BAB factor, charge four times more per unit of beta relative to funds introduced in unconstrained periods.

The results for the ICR measure are reported in columns (3) and (4), and are similar to the results based on the BAB factor. The coefficient on the interaction between market beta and the indicator for constrained periods equals 0.61, while the coefficient on the interaction between market beta and the indicator for unconstrained periods equals 0.24. The difference between the coefficients is statistically significant at about the 5% level. According to the ICR-based results, funds introduced in constrained periods charge between two and three times more per each unit of beta.

Finally, we repeat the analysis using the LCT measure and present the results in columns (5) and (6). The coefficient on the interaction between market beta and the indicator for constrained periods is almost twice as large as the coefficient on the interaction between market beta and the indicator for unconstrained periods. The difference between the coefficients is not significant in this case, potentially reflecting that LCT is a measure of fund managers' leverage constraints and slightly less indicative of fund investors' constraints.

In sum, the evidence on the time variation in borrowing constraints additionally supports Hypothesis 2. In more constrained periods, investors pay more for the same beta relative to less constrained periods.

4.2.3 Evidence on Time Variation in Demand

Our theory suggests that the relation between beta and fees is driven by increased demand for high-beta funds. To evaluate the importance of the demand channel, we examine the effects of time variation in borrowing constraints on fund flows. In particular, we test whether high-beta funds experience higher net flows immediately after borrowing constraints tighten. As fund flows vary significantly over time for a given fund as opposed to fees, which show almost no time variation, we can take full advantage of within-fund variation in flows for these tests. We set up a panel regression at the fund share class level of the form:

$$Netflow_{i,t+1} = \gamma_i + \gamma_t + \lambda\,(Beta_{it} \times Constrained_t) + \\ + \theta\,(Beta_{it} \times Unconstrained_t) + \rho X_{it} + e_{i,t+1}, \quad (12)$$

where $Netflow_{i,t+1}$, defined as $\frac{TNA_{i,t+1}-TNA_{i,t}(1+R_{i,t+1})}{TNA_{i,t}}$, is the net fund flow for fund i in month $t+1$, γ_i and γ_t are fund and month fixed effects, and X_{it} is the set of fund-level time-varying control variables from the main specification. Standard errors are double-clustered by fund family and month. The specification of constrained and unconstrained periods is in line with the previous section, where we evaluate the effects of borrowing constraints on fees across funds.

We report our findings in Table 7. The results consistently support the leverage demand channel, strengthening the evidence from Table 6. Higher-beta funds exhibit higher net flows in constrained periods, as measured by the BAB factor and the ICR measure (columns (1)–(4)). When fund beta increases from 1 to 1.7 in constrained time periods, the fund experiences an additional increase in net flows of 0.7–1.1 percentage points, relative to unconstrained time periods. This effect equals 16%–26% of the standard deviation of net flows, indicating that the economic magnitude is non-negligible. Consistent with the results on fees, when we measure the tightness of borrowing constraints using the LCT factor, the difference between the coefficients for constrained and unconstrained times is positive but not statistically significant (column (6)).[18]

In sum, our findings in Table 6 and Table 7 suggest that investors' demand for leverage drives both prices and quantities in the asset management market. Specifically, the leverage demand effect leads to cross-fund dispersion in prices (fund fees) and time-series fluctuation of quantities (fund AUM).

4.3 Implications for Fund Net Performance

We finally test Hypothesis 3 and examine the effects of leverage constraints on fund net performance. Our model suggests that the presence of leverage constraints is associated with reduced net alphas since investors pay fees not only for performance but also for embedded leverage. Consequently, we expect fund net alpha to decline in beta faster than gross alpha in the cross-section of funds for beta larger than one.

We conduct a portfolio analysis to test this prediction. We sort funds with betas larger than one into five equally-weighted portfolios according to the funds' beta and calculate mean gross and net alphas as well as mean gross and net returns for these portfolios.[19] The results for the share-class-level dataset are presented in Panel A of Table 8. Consistent with our regression results, fees are steadily increasing with beta across fund portfolios (column

[18]Our result that higher-beta funds exhibit higher net flows in constrained periods compared to unconstrained periods holds regardless of whether we control for fund performance or not. Not controlling for performance, however, results in strongly downward-biased coefficients on beta due to a classic omitted variable problem. Specifically, funds with high alphas attract higher flows (Chevalier and Ellison (1997), Sirri and Tufano (1998)), and funds' gross alpha declines in beta due to a relatively flat security market line, as we discuss in the next section. Therefore, omitting alpha in the fund flows regressions projects this relation on beta and leads to negative beta coefficients in both constrained and unconstrained times (columns (1), (3), and (5)).

[19]Naturally, we focus on active mutual funds for the analysis of fund performance, excluding passive mutual funds and ETFs from this analysis.

(1)). The difference in fees between the high-beta portfolio and the low-beta portfolio is equal to 0.23%.

We report the average gross CAPM alphas in column (2). Gross alpha is declining with beta in line with a relatively flat security market line in the asset market (see Black, Jensen, and Scholes (1972), Frazzini and Pedersen (2014)). The difference in gross alphas between high-beta funds and low-beta funds equals -0.37%, but it is not statistically significant at the 10% level. At the same time, net alpha declines with beta one-and-a-half to two times as fast as gross alpha (column (3)). The difference in net alphas between the high-beta portfolio and the low-beta portfolio equals -0.60%, statistically significant at the 5% level. The results are very similar for the fund-level analysis presented in Panel B.

These findings suggest that two mechanisms can jointly explain why net performance declines with beta: (1) the leverage demand effect of fund investors presented in this paper, which drives the increase in fees; and (2) the asset market mechanism which drives the decline in gross alpha (e.g., Frazzini and Pedersen (2014)). Both mechanisms are in line with Hypothesis 3, and they generate approximately equally-sized effects on the observed decline in net alpha.

Finally, the results in columns (4) and (5) show that high-beta funds have higher average excess returns. High-beta funds are therefore indeed attractive to leverage-constrained risk-seeking investors, even though the risk-return relation inherited from the asset market is flatter than predicted by the CAPM. This is again in line with the BAB case in our calibrated model: a flatter security market line slightly weakens the relation between beta and fees for funds with betas greater than one, but this effect is not large enough to eliminate the relation.

5 The Role of Fund Investment Practices and Trading Costs

Our theory suggests that high fees for high-beta funds stem from the investors' willingness to pay for leverage, as strongly supported by our empirical findings. As a complementary channel, fees could be driven by the asset managers' costs of providing high embedded leverage in terms of beta. Intuitively, asset managers can lever up their portfolios in two broad ways: (1) investing in high-beta stocks; and (2) engaging in alternative investment practices such as borrowing capital directly, trading derivatives, or using short-selling. Either of these alternatives could result in higher asset management costs compared to funds that do not use alternative practices or mostly trade lower-beta stocks. To account for the possibility of such cost-side effects, we explore the relation of funds' investments practices and trading costs to embedded leverage and fees, and examine how these parameters interact with our results.

5.1 Data from N-SAR Filings

We obtain information on fund investment practices from N-SAR filings, which are required for registered investment management companies. The filings are made available by the SEC in a standardized electronic format through the EDGAR database. Item 70 of the N-SAR form provides detailed information on whether the fund has engaged in various investment practices during the reporting period. We collect these filings using an automated scraping algorithm and match them by the fund name to our main fund-level sample. The matching of fund names is done algorithmically and is validated by manual checks.[20]

Overall, 70% of the funds in our main sample have at least one N-SAR filing matched. N-SAR forms are filed semiannually, and we match the last month of the reporting period to our main sample.[21] We ultimately have 26,831 fund-month observations matched over the period 1995–2016. For the matched funds, we observe one N-SAR filing record per year on average, suggesting a fund-month matching rate of approximately 50%. We also confirm that our baseline results on beta and fees are strongly robust in the matched subsample.

[20] We match fund names based on the Levenshtein distance, a leading string matching metric in computer science. While the algorithm assigns a match to every fund name in principle, we treat entries with a matching score below 95 (out of 100) as unmatched. This strategy ensures that there are no false positive matches. An example for a match with a score of 95 is 'Phoenix Strategic Equity Series Fund: Phoenix-Seneca Equity Opportunities Fund' in CRSP vs. 'PHOENIX STRATEGIC EQUITY SERIES FUND: PHOENIX EQUITY OPPORTUNITIES FUND' in the N-SAR filings.

[21] The N-SAR filings do not specify during which particular months of a reporting period a certain investment practice was used, but funds do not change their investment practices very frequently. For the investment practices of interest, as defined below, the 1-period (half-year) autocorrelation is 0.82, and fund fixed effects explain 71% of the variation. Our results are almost identical when we match the information from an N-SAR filing to *all* months of the reporting period.

We focus on a number of selected investment practices which are associated with leverage. The fund's answers to the questions (Qs) in the N-SAR filings precisely reveal whether the fund engages in these alternative practices. In particular, we follow Warburton and Simkovic (2019) and collect information on whether the fund: (1) borrows money (Q 70.O); (2) engages in short-selling (Q 70.R); (3) trades in options on individual equities or stock indices (Qs 70.B and 70.D); or (4) trades in stock index futures or options on stock index futures (Qs 70.F and 70.H). We create an indicator variable that equals one if the fund engages in any of these alternative investment practices, as well as similarly defined indicator variables for each practice separately.

5.2 Fund Investment Practices, Trading Costs, and Beta

We present the summary statistics for fund investment practices in Panel A of Table 9. Only 30% of the sample funds employ any alternative investment practice. The most common practices are trading in stock index futures (17%), borrowing money (8%), and trading options on equities (7%). The fraction of funds engaged in each of the other practices is below 3%.

Furthermore, the funds with betas greater than one do not engage in alternative investment practices more frequently than the rest of the funds. The fraction of high-beta funds engaged in any of these practices equals 29%, which is approximately the same as in the entire sample. A similar pattern holds for each investment practice separately: the difference in the fraction of funds engaged in a practice between the entire sample and the sample of high-beta funds is never above 1%. These results show that most high-beta funds are not especially reliant on borrowing, usage of derivatives, or short-selling, suggesting that they achieve high beta by holding high-beta stocks.

As an additional measure for the presence of alternative practices, we also examine the difference between the fund's beta and the weighted average beta of its stock holdings. A positive difference indicates that the fund uses instruments other than stocks to lever up its portfolio. Based on this idea, we construct an indicator variable which equals one if the difference between the fund's beta and the beta of the stock portfolio is larger than 0.05. We require the difference to be slightly larger than zero due to potential errors in the estimation of betas.[22] Using quarterly holdings data from Thomson Reuters, we first construct the stock portfolio beta as the weighted average across all the individual stocks held in the fund portfolio in the last month of each quarter (i.e., in March, June, September, and December), and then we calculate the difference between the fund beta and the portfolio beta in each of those corresponding months. The results in Panel A of Table 9 show that only 25% of high-beta funds have betas larger than the betas of their stock portfolios. This finding is again in line with the prevalent reliance on high-beta stocks for obtaining high-beta portfolios.

Motivated by these statistics, we explore the relation between stock trading costs and fund beta. Since our findings imply that most of the high-beta funds obtain their betas by investing in high-beta stocks, differences in stock trading costs may affect the relation between beta and fees. We use the proportional effective spread (PES) as a proxy for the stock's trading costs (Lesmond, Schill, and Zhou (2004), Hasbrouck (2009), and Novy-Marx and Velikov (2016)). In each reporting month (again the last month of each quarter) and for each stock in the fund's portfolio, we calculate the daily stock PES using the closing prices as

$$PES_{it} = \frac{2|Price_{it} - 0.5 \times (Bid_{it} + Ask_{it})|}{Price_{it}}. \tag{13}$$

We calculate the monthly stock PES as a daily average and the fund's PES as a market-capitalization-weighted PES of the stocks held in the fund's portfolio. The results in Panel A of Table 9 show that the average stock portfolio PES across our sample funds equals 0.19%, while the average high-beta fund has a PES of 0.17%. This initial descriptive evidence suggests that high-beta funds do not incur higher stock trading costs.

Finally, we formally examine the relation of investment practices and trading costs to fund beta within the sample of high-beta funds. We regress fund beta on our proxies for alternative investment practices or on the fund PES. All the specifications include the same set of control variables and fixed effects as in our main specifications for fees. Standard errors are double-clustered by fund family and month.

We report the results in Panel B of Table 9. Overall, the effects are economically negligible. The funds which engage in borrowing, usage of derivatives, or short-selling, as

[22]The stock betas are estimated using the procedure described in Section 3.2.

reported in their N-SAR filings, have betas lower by 0.01, relative to the funds which do not engage in these practices (column (1)). A positive difference between the fund's beta and the beta of its stock portfolio is associated with a tiny increase of 0.03 in fund beta (column (2)). An increase of one standard deviation in stock portfolio PES (which is 0.20%) is associated with an increase of 0.026 in fund beta (column (3)). These results again indicate that the fund's engagement in alternative investment practices or its stock trading costs are largely unrelated to the fund's beta.

In sum, our findings imply that most of the high-beta funds rely on high-beta stocks to obtain their betas. Moreover, these funds are not more likely to engage in borrowing, usage of derivatives, or short-selling, and they do not face higher stock trading costs.

5.3 Effects on the Relation between Beta and Fees

Finally, we examine the impact of fund investment practices and trading costs on the relation between beta and fees. If this relation is driven by fund management costs, it should be affected when conditioning on the investment practice or stock trading cost variables. Importantly, we do not assume a priori which investment practices are more expensive than others. For example, it is unclear whether trading derivatives is more expensive than trading high-beta stocks. Instead, we argue that if such a difference exists and it strongly affects fund management costs, this difference may drive the effect of beta on fees, beyond our baseline demand-driven effect.

To conduct this analysis, we split the sample of funds with betas greater than one into subsamples based on our proxies for the alternative investment practices, or based on the fund PES. We estimate our main specification in each subsample and compare the coefficients on beta across the subsamples. The results in Table 10 show that the relation between beta and fees does not depend on fund investment practices and trading costs. In columns (1) and (2), we compare the funds that employ alternative investment practices to the rest of the funds. The coefficients on beta are positive and statistically significant in both subsamples. The p-value of the Wald test for comparison between the coefficients across the subsamples equals 0.51, indicating that the difference is statistically indistinguishable from zero (column (3)). We obtain similar results when we compare the funds which have betas higher than their portfolio betas to the rest of the funds (columns (4)–(6)), as well as comparing the funds with above-the-median PES to the funds with below-the-median PES (columns (7)–(9)).

Overall, these findings indicate that the relation between beta and fees does not depend on investment practices and stock trading costs. The results are consistent with the evidence from Table 9: since there is no strong relation between fund beta and investment practices, the relation between beta and fees is also unlikely to be affected. We conclude that the distinctive relation of fund betas and fees highlighted in this paper is primarily and consistently shaped by investors' demand for leverage.

6 Conclusion

In this paper, we develop and test a theory suggesting that investors pay fees for leverage provided by their asset managers. If investors face borrowing constraints and are limited in making leveraged investments on their own, they seek for managers to obtain the desired leveraged returns. Based on this insight, we theoretically derive an asymmetric relation between beta and fees and show that this relation varies with the tightness of leverage constraints. The empirical evidence from the U.S. equity mutual funds provides strong support for the model's predictions: fees vary across funds, investors, and market conditions in a manner consistent with the leverage-based explanation.

Our results shed light on the well-known poor performance of asset managers who charge fees that are significantly higher than the managers' risk-adjusted returns. We propose that high-beta funds provide an additional service to their borrowing-constrained investors. The investors can lever up their portfolios through the asset manager and pay fees for the embedded leverage irrespective of the fund performance. Consequently, fund gross alpha may not fully capture the full range of services provided by asset managers. Many high-beta funds appear as "underperforming" net-of-fees while their investors can actually improve their welfare by gaining access to leverage.

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Figure 1: Risk Aversion, Fund Beta, and Willingness to Pay

This figure presents constellations of investor i's risk aversion γ_i and fund j's fee ϕ_j for which j is preferred over the market index fund with $\beta_M = 1$ and $\phi_M = 0$. The blue region presents the relation for a fund with $\beta_j = 1.1$, the yellow region presents the relation for a fund with $\beta_j = 1.3$. The left plot describes investors who face strict borrowing constraints, the right plot presents the case of less constrained investors with $l = 2$. The dashed lines stand for the ϕ_j value above which condition (4) is fulfilled, i.e., in which the region is linear as in Proposition 2. Parameters are set according to the CAPM case in Table 1.

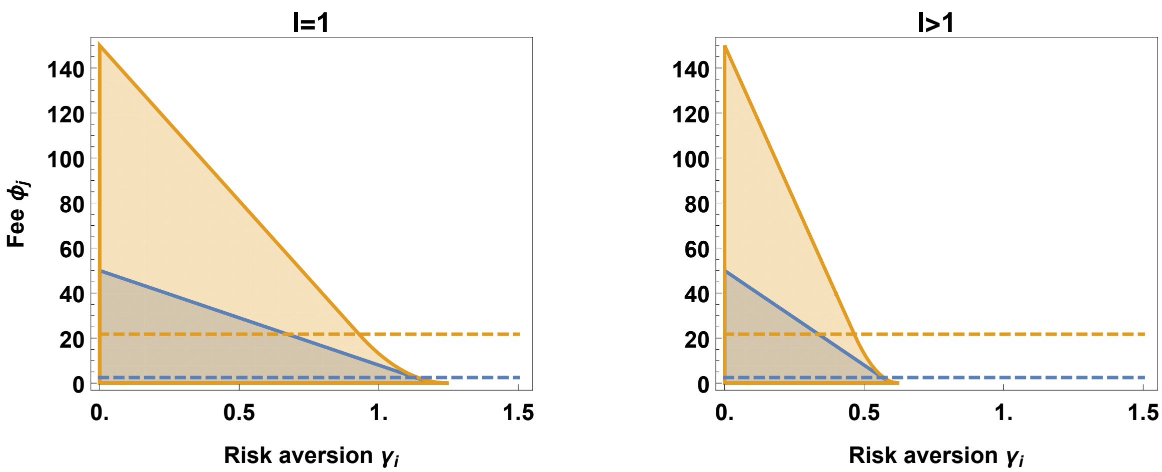

Figure 2: Distribution of Investors across Funds

This figure illustrates how investors sort across asset managers based on their risk aversion γ_i, given four managers with $\beta > 1$, the market index fund with $\beta_1 = \beta_M = 1$, and possible additional funds with $\beta < 1$. Fund fees are set exemplarily to $(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5) = (0, 2.5, 25, 65, 120)$ basis points. All the investors face strict borrowing constraints ($l = 1$). Further parameters are set according to the CAPM case in Table 1.

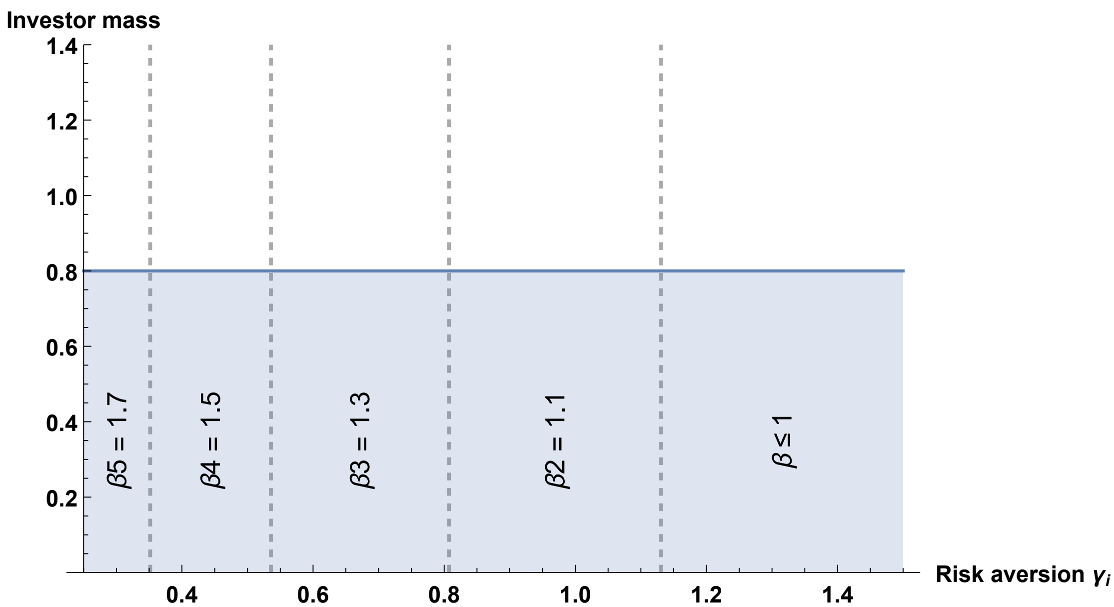

Figure 3: The Theoretical Relationship between Beta and Fees

This figure presents the relation between fund betas and fees as predicted by our model. The blue line stands for a scenario with "few" funds (i.e., two $\beta > 1$-funds, the market ETF, and an arbitrary number of $\beta < 1$-funds) where the parameters are set according to the "less constrained" scenario in Table 1. The yellow line repeats the scenario with "few" funds but for the case when all the investors face strict borrowing constraints ($l = 1$). The green line describes a setting with "many" funds according to Table 1 for which we numerically solve for the equilibrium. The orange line results from the BAB case, while all other lines employ the CAPM case. Hollow circles indicate that in the CAPM case, the fee for any $\beta < 1$ fund is exactly zero. In all scenarios, fund fees result endogenously from the model equilibrium.

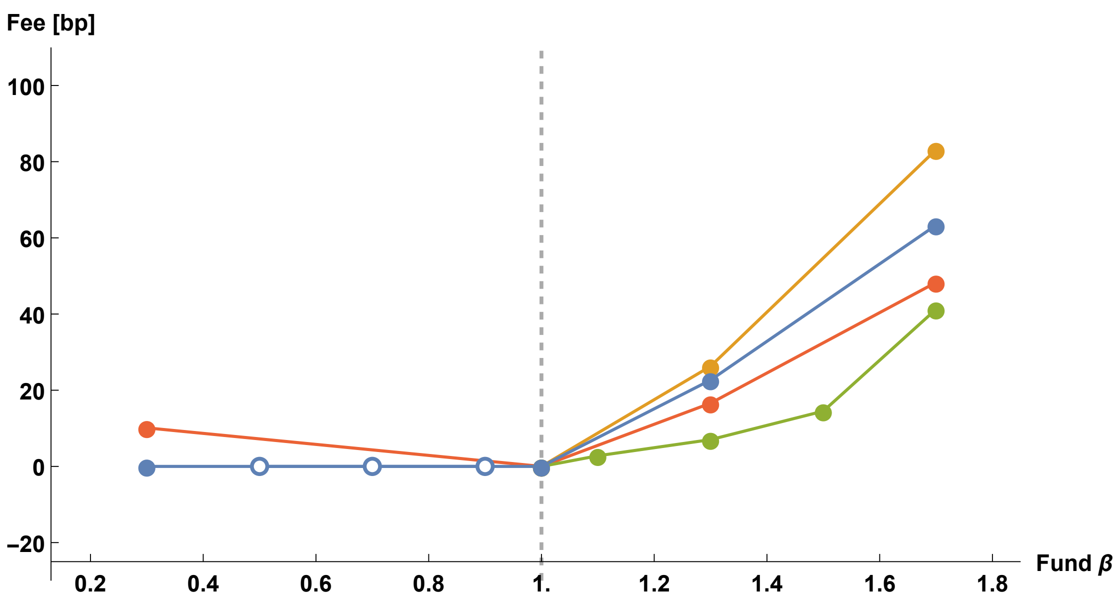

Figure 4: Distribution of Fund Beta

This figure presents the empirical distribution of funds across market betas. The bars show the fraction of funds for each level of beta. *Beta* is an estimate of the slope from the market model for fund returns.

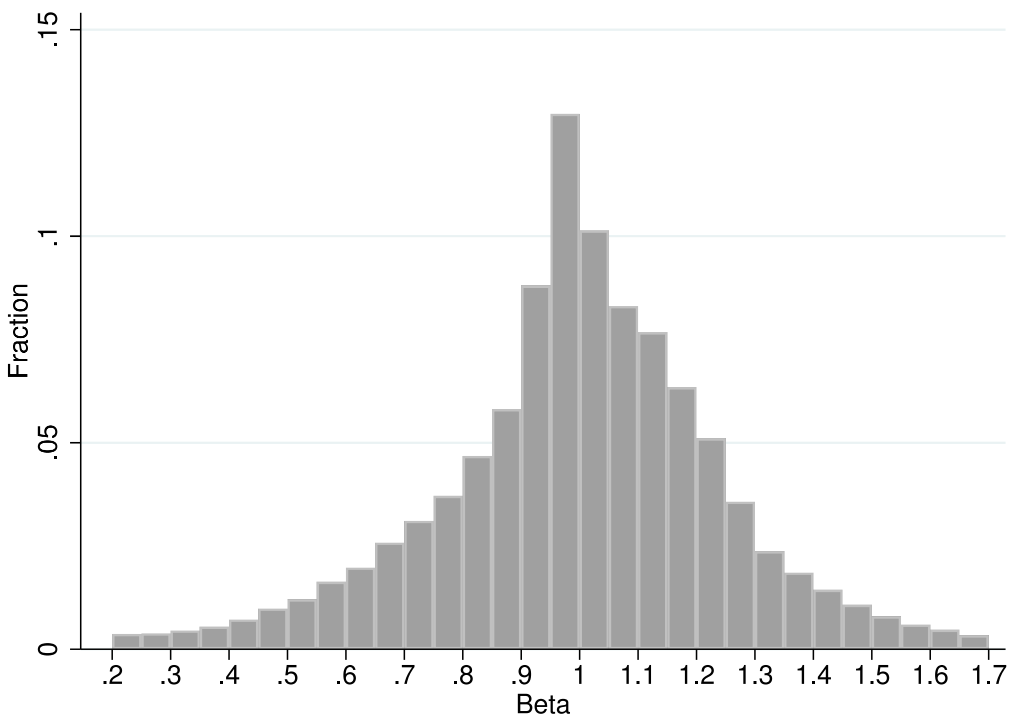

Figure 5: The Empirical Relationship between Beta and Fees

This figure presents the binscatter plot of residual fees against fund betas separately for funds with betas larger than one and smaller than one. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Residual fee* is estimated in two steps: First, we regress the fee on all the control variables and fixed effects. Second, we calculate the residual fee as the original fee minus the predicted value based on the estimation in the first step. The shaded areas represent 95% confidence intervals.

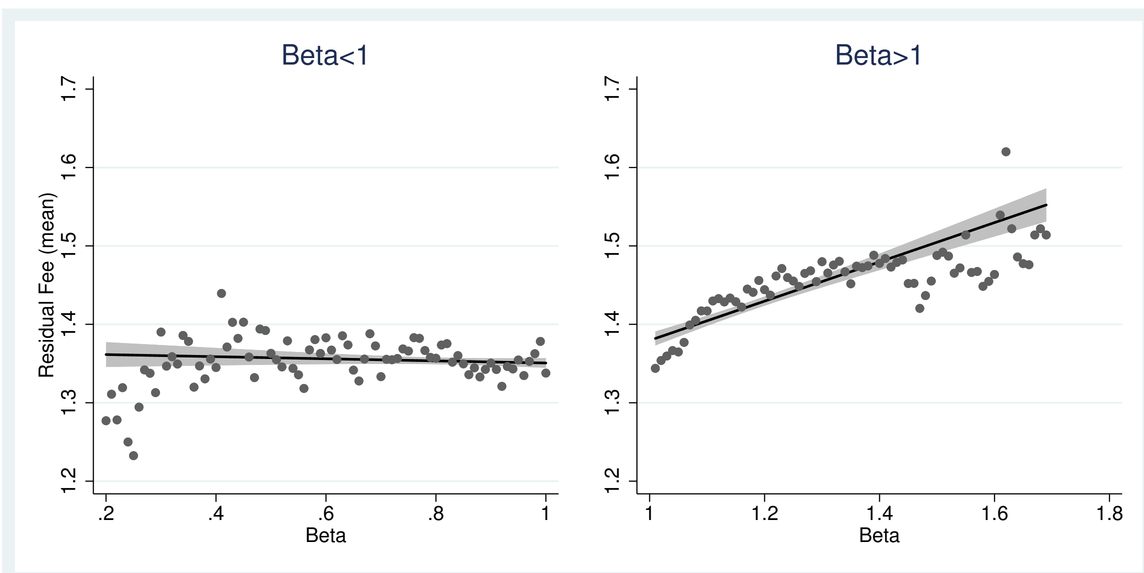

Table 1: Model Parameters, Borrowing-Constraint Scenarios, and Fund Betas

This table presents the parameters we use to calibrate the model for different scenarios. Parameters describing the expected return and volatility of the stock market are set to the standard values of 5% and 20% per year, respectively. We consider a baseline CAPM case and a betting-against-beta (BAB) case with a flatter security market line. The investors' absolute risk aversion in our calibrated model is uniformly distributed between 0.25 and 1.50. For the investors' borrowing constraints, we consider a "more constrained" scenario in which all investors are strictly borrowing-constrained, and a "less constrained" scenario in which 25% of investors are strictly constrained and 75% can obtain a leverage of 2. We analyze scenarios with "few funds", i.e., two funds with beta greater than one in addition to the market ETF and a fund with beta smaller than one, and "many funds" with six funds overall (four of them with beta greater than one).

Parameter		Value	
Stock market			
Expected stock market return	μ_M	0.05	
Stock market volatility	σ_M	0.2	
		CAPM	BAB
Betting-against-beta parameter	ξ	0.0	0.015
Fund investors			
Highest absolute risk aversion	$\overline{\Gamma}$	1.50	
Lowest absolute risk aversion	$\underline{\Gamma}$	0.25	
		More constrained	Less constrained
Mass of strictly borrowing-constr. investors ($l = 1$)	ψ	1	0.25
Max. leverage for less borrowing-constrained investors	$\bar{l}$	—	2
Fund characteristics			
		Few funds	Many funds
	β_0	0.3	0.3
	β_1	1.0	1.0
Fund betas	β_2	1.3	1.1
	β_3	1.7	1.3
	β_4	—	1.5
	β_5	—	1.7

Table 2: Summary Statistics

This table presents summary statistics for the sample of fund-month observations over the period 1991–2016 at the fund share class level (Panel A), at the fund level (Panel B), and at the fund share class level at the time of fund launch (Panel C). The fund characteristics are from the CSRP mutual fund database. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* and *Net CAPM alpha* are annualized estimates of the intercept from the market models for fund gross returns and fund net returns, respectively. *Log(TNA)* is the natural logarithm of fund total net assets. *Log(Age)* is the natural logarithm of fund age in months. *(0,1) Passive fund* indicator equals one if a fund is passively managed. *(0,1) ETF* indicator equals one if a fund is an ETF. *(0,1) Retail fund* indicator equals one if a share class is offered to retail investors. *Net flow* is the monthly net fund flow. *Number of funds per beta bin* is the number of funds (in thousands) with the value of beta falling into each 0.1 bin (e.g., funds with betas between 0.8–0.9 are in a bin, funds with betas between 1.1–1.2 are in another bin, etc.) in a specific month. *HHI per beta bin* is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. The last columns of Panels A and B report the R^2 of the regressions of variables on fund share class fixed effects or fund fixed effects, respectively.

Panel A: Fund Share Classes	N	Mean	SD	Within SD	5%	25%	50%	75%	95%	R^2 - fund share class FE
Fee (%)	989,552	1.57	0.75	0.03	0.37	0.99	1.52	2.15	2.76	0.96
Beta	989,552	1.00	0.21	0.04	0.61	0.90	1.00	1.13	1.35	0.70
Gross CAPM alpha (%)	989,552	1.33	4.95	0.12	-4.88	-1.12	0.85	3.27	9.59	0.39
Net CAPM alpha (%)	989,552	-0.26	4.90	0.13	-6.63	-2.62	-0.63	1.63	7.81	0.39
Log(TNA)	989,552	4.15	2.34	0.36	0.09	2.65	4.27	5.80	7.76	0.88
Log(Age)	989,552	4.81	0.44		4.15	4.46	4.77	5.12	5.56	
(0,1) Passive fund	989,552	0.07	0.26		0.00	0.00	0.00	0.00	1.00	
(0,1) ETF	989,552	0.02	0.15		0.00	0.00	0.00	0.00	0.00	
(0,1) Retail fund	989,552	0.66	0.47		0.00	0.00	1.00	1.00	1.00	
Net flow (%)	958,242	-0.63	4.44		-6.56	-2.18	-0.76	0.64	5.90	
N of funds per beta bin	989,552	1.14	0.69		0.15	0.49	1.13	1.84	2.11	
HHI per beta bin	989,552	0.03	0.03		0.01	0.01	0.02	0.03	0.07	

Table 2: Summary Statistics (continued)

Panel B: Funds	N	Mean	SD	Within SD	5%	25%	50%	75%	95%	R^2 - fund FE
Fee (%)	439,539	1.38	0.69	0.07	0.27	0.93	1.30	1.84	2.54	0.90
Beta	439,539	1.00	0.24	0.05	0.56	0.87	1.00	1.14	1.39	0.66
Gross CAPM alpha (%)	439,539	1.45	5.52	1.88	-5.57	-1.14	0.95	3.60	10.72	0.34
Net CAPM alpha (%)	439,539	0.04	5.47	1.87	-7.16	-2.46	-0.34	2.12	9.17	0.34
Log(TNA)	439,539	5.64	1.86	0.37	2.54	4.37	5.68	6.95	8.65	0.85
Log(Age)	439,539	4.84	0.47		4.13	4.48	4.84	5.18	5.64	
(0,1) Passive fund	439,539	0.10	0.30		0.00	0.00	0.00	0.00	1.00	
(0,1) ETF	439,539	0.05	0.22		0.00	0.00	0.00	0.00	1.00	
Net flow (%)	426,277	-0.37	3.83		-5.44	-1.71	-0.55	0.67	5.49	
N of funds per beta bin	439,539	0.37	0.22		0.05	0.17	0.31	0.58	0.69	
HHI per beta bin	439,539	0.05	0.05		0.01	0.03	0.04	0.05	0.12	

Panel C: Funds at Launch	N	Mean	SD	5%	25%	50%	75%	95%
Fee (%)	11,520	1.60	0.83	0.29	0.98	1.50	2.22	2.94
Beta	11,520	0.98	0.25	0.52	0.85	0.99	1.12	1.41
Gross CAPM alpha (%)	11,520	1.32	6.05	-6.22	-1.46	0.80	3.62	11.56
Net CAPM alpha (%)	11,520	-0.27	6.00	-8.06	-2.89	-0.61	1.93	9.69
Log(TNA)	11,520	1.56	2.50	-2.30	-0.22	1.71	3.43	5.46
(0,1) Passive fund	11,520	0.08	0.26	0.00	0.00	0.00	0.00	1.00
(0,1) ETF	11,520	0.04	0.19	0.00	0.00	0.00	0.00	1.00
(0,1) Retail fund	11,520	0.59	0.49	0.00	0.00	1.00	1.00	1.00

Table 3: Relation between Fund Market Beta and Fund Fees

This table reports the results from regressing mutual fund fees on fund market beta and fund characteristics separately for funds with betas larger than one and smaller than one. Columns (1)–(4) report the results for the fund-share-class-level sample, and columns (5)–(8) report the results for the fund-level sample. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* is an annualized estimate of the intercept from the market model for fund gross returns. *Log(TNA)* is the natural logarithm of fund total net assets. *Log(Age)* is the natural logarithm of fund age in months. *(0,1) Passive fund* indicator equals one if a fund is passively managed. *(0,1) ETF* indicator equals one if a fund is an ETF. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Share class level				Fund level			
	Beta>1		*Beta*<1		*Beta*>1		*Beta*<1	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Beta	0.44***	0.48***	-0.09*	0.01	0.33***	0.35***	-0.07	-0.02
	(0.06)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.06)
Gross CAPM alpha		0.02***		0.01***		0.01***		0.00***
		(0.00)		(0.00)		(0.00)		(0.00)
Log(Age)	0.20***	0.21***	0.17***	0.17***	0.07**	0.07***	0.10***	0.10***
	(0.04)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Log(TNA)	-0.08***	-0.09***	-0.06***	-0.06***	-0.06***	-0.06***	-0.04***	-0.04***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
(0,1) Passive fund	-0.51***	-0.52***	-0.60***	-0.60***	-0.51***	-0.52***	-0.60***	-0.60***
	(0.09)	(0.09)	(0.06)	(0.06)	(0.08)	(0.08)	(0.06)	(0.06)
(0,1) ETF	-0.27	-0.24	0.11	0.09	-0.35	-0.34	0.06	0.04
	(0.25)	(0.25)	(0.12)	(0.12)	(0.23)	(0.23)	(0.15)	(0.15)
Observations	511,810	511,810	476,797	476,797	219,912	219,912	218,872	218,872
R-squared	0.47	0.48	0.46	0.47	0.69	0.69	0.66	0.66
Fund family fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 4: Relation between Fund Market Beta and Fund Fees: Robustness Checks

This table reports the results of robustness checks for the main tests presented in Table 3. Panel A reports the results from regressing fund fees on fund market beta and measures of the intensity of alternative fund offerings. Panel B reports the results from regressing fund fees on fund market beta separately for *Broker-sold* and *Direct-sold* funds. Panel C reports the results from regressing fund fees on fund market beta and style fixed effects. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Number of funds per beta bin* is the number of funds with the value of beta falling into each 0.1 bin (e.g., funds with betas between 0.8–0.9 are in a bin, funds with betas between 1.1–1.2 are in another bin, etc.) in a specific month. *HHI per beta bin* is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. A fund share class is considered *Direct-sold* if it charges no front or back load, and has an annual distribution fee ("12b-1" fee) of no more than 25 basis points; otherwise it is considered *Broker-sold*. *Style fixed effects* are defined based on the fund Lipper classification. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. Tables A1–A3 in the appendix present the detailed results for these tests. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Share class level		Fund level	
	(1)	(2)	(3)	(4)
	Beta>1	*Beta*<1	*Beta*>1	*Beta*<1
	Coefficient on *Beta*			
Panel A: Controlling for fund offerings				
Add *N of funds per beta bin*	0.33***	-0.09	0.28***	-0.00
	(0.07)	(0.07)	(0.07)	(0.07)
Add *HHI per beta bin*	0.52***	0.04	0.39***	0.03
	(0.06)	(0.05)	(0.05)	(0.06)
Panel B: Distribution channels (subsamples)				
Within *Broker-sold*	0.45***	0.06		
	(0.06)	(0.06)		
Within *Direct-sold*	0.39***	0.09		
	(0.04)	(0.06)		
Panel C: Fund styles				
Add *Style fixed effects*	0.27***	0.02	0.21***	0.04
	(0.06)	(0.06)	(0.05)	(0.06)

Table 5: Relation between Fund Market Beta, Fund Fees, and Investor Type

This table reports the results from regressing fund fees on fund market beta and its interactions with indicators for retail and institutional share classes. All the regressions are estimated for funds with betas larger than one. Columns (1) and (2) present the results for the fund-share-class-level sample, and columns (3) and (4) present the results for the fund launch sample. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* is an annualized estimate of the intercept from the market model for fund gross returns. *(0,1) Retail fund* indicator equals one if a share class is offered to retail investors. *(0,1) Institutional* indicator equals one if a share class is offered to institutional investors. The p-values for tests of differences between the coefficients are reported. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Share class level		Fund launch	
	(1)	(2)	(3)	(4)
*(0,1) Retail * Beta*	0.44***	0.46***	0.43***	0.44***
	(0.06)	(0.05)	(0.09)	(0.09)
*(0,1) Institutional * Beta*	0.30***	0.34***	0.22***	0.24***
	(0.07)	(0.06)	(0.06)	(0.06)
(0,1) Retail	0.73***	0.73***	0.65***	0.65***
	(0.08)	(0.08)	(0.12)	(0.12)
Gross CAPM alpha		0.01***		0.01***
		(0.00)		(0.00)
Tests for differences between coefficients				
*(0,1) Retail * Beta – (0,1) Institutional * Beta*	0.14**	0.12*	0.21**	0.20**
p-value	0.05	0.06	0.02	0.02
Observations	476,797	476,797	5,186	5,186
R-squared	0.68	0.68	0.70	0.71
Control variables	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes

Table 6: Relation between Fund Market Beta, Fund Fees, and Tightness of Borrowing Constraints

This table reports the results from regressing fund fees on fund market beta and its interactions with measures of borrowing constraint tightness. The measures include the BAB measure from Frazzini and Pedersen (2014), the ICR measure from He, Kelly, and Manela (2017), and the LCT measure from Boguth and Simutin (2018). All the regressions are estimated for funds with betas larger than one at the time of fund launch. The sample consists of months when the measure of tightness is either in the first quartile or in the fourth quartile of its distribution across time. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* is an annualized estimate of the intercept from the market model for fund gross returns. *(0,1) Constrained* indicator is defined for each measure separately and equals one if the BAB or ICR measures are in the first quartile of their distributions across time, and if the LCT measure is in the fourth quartile of its distribution across time. *(0,1) Unconstrained* equals one minus *(0,1) Constrained*. The p-values for tests of differences between the coefficients are reported. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*						
Measure of borrowing constraint tightness	BAB		ICR		LCT	
	(1)	(2)	(3)	(4)	(5)	(6)
*(0,1) Constrained * Beta*	0.54***	0.55***	0.61***	0.66***	0.31**	0.30**
	(0.13)	(0.13)	(0.15)	(0.16)	(0.13)	(0.14)
*(0,1) Unconstrained * Beta*	0.12	0.14	0.24***	0.22**	0.17	0.18
	(0.10)	(0.11)	(0.08)	(0.09)	(0.14)	(0.14)
Gross CAPM alpha		0.01***		0.01*		0.00
		(0.00)		(0.00)		(0.00)
Tests for differences between coefficients						
*(0,1) Constrained * Beta – (0,1) Unconstrained * Beta*	0.42**	0.41**	0.37*	0.44**	0.14	0.12
p-value	0.03	0.04	0.06	0.04	0.50	0.53
Observations	2,616	2,616	2,150	2,150	2,462	2,462
R-squared	0.72	0.72	0.74	0.74	0.70	0.70
Control variables	Yes	Yes	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

Table 7: Relation between Fund Market Beta, Net Fund Flows, and Tightness of Borrowing Constraints

This table reports the results from regressing net fund flows on fund market beta and its interactions with measures of borrowing constraint tightness. The measures include the BAB measure from Frazzini and Pedersen (2014), the ICR measure from He, Kelly, and Manela (2017), and the LCT measure from Boguth and Simutin (2018). All the regressions are estimated for funds with betas larger than one. The sample consists of months when the measure of tightness is either in the first quartile or in the fourth quartile of its distribution across time. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* is an annualized estimate of the intercept from the market model for fund gross returns. *(0,1) Constrained* indicator is defined for each measure separately and equals one if the BAB or ICR measures are in the first quartile of their distributions across time, and if the LCT measure is in the fourth quartile of its distribution across time. *(0,1) Unconstrained* equals one minus *(0,1) Constrained*. The p-values for tests of differences between the coefficients are reported. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Net Flow*						
Measure of borrowing constraint tightness	BAB		ICR		LCT	
	(1)	(2)	(3)	(4)	(5)	(6)
*(0,1) Constrained * Beta*	-0.69**	0.72**	-0.73**	0.99***	-0.69**	0.89**
	(0.31)	(0.34)	(0.37)	(0.31)	(0.31)	(0.38)
*(0,1) Unconstrained * Beta*	-1.52***	-0.28	-1.46***	-0.64*	-1.34***	0.34
	(0.34)	(0.38)	(0.34)	(0.36)	(0.31)	(0.38)
Gross CAPM alpha		0.17***		0.17***		0.17***
		(0.01)		(0.00)		(0.01)
Tests for differences between coefficients						
*(0,1) Constrained * Beta – (0,1) Unconstrained * Beta*	0.83***	1.00***	0.73*	1.63***	0.65**	0.55
p-value	0.008	0.003	0.09	0.0001	0.04	0.124
Observations	244,136	244,136	240,159	240,159	193,372	193,372
R-squared	0.16	0.17	0.17	0.19	0.18	0.20
Control variables	Yes	Yes	Yes	Yes	Yes	Yes
Fund fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

Table 8: Average Returns, CAPM Alphas, and Fees for Fund Portfolios Sorted by Market Beta

This table reports average returns, CAPM alphas, and mutual fund fees for five equally weighted mutual fund portfolios sorted by their market betas. All the funds have betas larger than one. Panel A presents the results from the fund share class sample and Panel B presents the results from the fund-level sample. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Gross CAPM alpha* and *Net CAPM alpha* are annualized estimates of the intercept from the market models for fund gross returns and fund net returns, respectively. *Net Return* is the annualized fund monthly return net-of-fees. *Gross Return* is the sum of the fund's *Net Return* and the fund's *Fee*. The t-statistics for tests of differences between the averages are reported. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively.

Panel A: Share class level	(1)	(2)	(3)	(4)	(5)
Quintile	*Fee*	*Gross CAPM Alpha*	*Net CAPM Alpha*	*Gross Return*	*Net Return*
(1) Low *Beta*	1.65	0.74	-0.92	8.17	6.49
(2)	1.71	0.80	-0.93	8.30	6.61
(3)	1.77	0.73	-1.07	8.73	6.91
(4)	1.81	0.44	-1.39	9.32	7.56
(5) High *Beta*	1.88	0.37	-1.52	9.93	8.02
High *Beta* – Low *Beta*	0.23***	-0.37	-0.60**	1.77	1.54
t-statistic	18.10	-1.47	-2.40	0.36	0.40
Panel B: Fund level					
(1) Low *Beta*	1.48	0.81	-0.67	8.22	6.75
(2)	1.55	0.97	-0.61	8.54	7.00
(3)	1.60	0.84	-0.80	8.90	7.29
(4)	1.66	0.65	-1.03	9.55	7.75
(5) High *Beta*	1.70	0.40	-1.30	9.42	7.87
High *Beta* – Low *Beta*	0.22***	-0.41	-0.63**	1.20	1.12
t-statistic	12.83	-1.57	-2.42	0.20	0.19

Table 9: Funds' Investment Practices, Trading Costs, and Market Beta

This table reports the information on fund investment practices and stock trading costs. Panel A reports the summary statistics for the investment practice variables obtained from the form N-SAR, the difference between fund beta and its stock holdings beta, as well as fund stock trading costs. Panel B presents the results from regressing fund betas on indicators for presence of the alternative investment practices and on fund stock trading costs. *Beta* is an estimate of the slope from the market model for fund returns. *(0,1) Alternative practices* indicator equals one if the fund engages in at least one of the activities such as borrowing money, short-selling, or trading options and futures. *(0,1) Options on equities* indicator equals one if the fund trades options on equities. *(0,1) Options on stock indices* indicator equals one if the fund trades options on stock indices. *(0,1) Stock index futures* indicator equals one if the fund trades stock index futures. *(0,1) Options on stock index futures* indicator equals one if the fund trades options on stock index futures. *(0,1) Borrowing money* indicator equals one if the fund borrows money. *(0,1) Short-selling* indicator equals one if the fund engages in short-selling. *(0,1) Fund beta>portfolio beta* indicator equals one if the difference between the fund beta and its stock holdings beta is larger than 0.05. *Stock portfolio PES* is a value-weighted average proportional effective spread of the fund holdings. All regressions are estimated for funds with betas larger than one, and they include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

Panel A: Summary Statistics	(1)	(2)	(3)	(4)
	Full Sample		*Beta*>1	
	Mean	N	Mean	N
(0,1) Alternative practices	0.30	26,831	0.29	13,076
(0,1) Options on equities	0.07	26,831	0.06	13,076
(0,1) Options on stock Indices	0.02	26,831	0.01	13,076
(0,1) Stock index futures	0.17	26,831	0.15	13,076
(0,1) Options on stock index futures	0.004	26,831	0.003	13,076
(0,1) Borrowing money	0.08	26,831	0.09	13,076
(0,1) Short-selling	0.03	26,831	0.02	13,076
(0,1) Fund beta>portfolio beta	0.20	49,756	0.25	23,675
Stock portfolio PES (%)	0.19	69,325	0.17	37,243

Panel B: Investment Practices, Transaction Costs, and Fund Beta (*Beta*>1)			
y = *Beta*	(1)	(2)	(3)
(0,1) Alternative practices	-0.01**		
	(0.00)		
(0,1) Fund beta>portfolio beta		0.03***	
		(0.01)	
Stock portfolio PES			0.13**
			(0.06)
Observations	26,789	49,695	69,245
R-squared	0.32	0.33	0.27

Table 10: Effects of Investment Practices and Trading Costs on the Relation between Fund Market Beta and Fund Fees

This table presents the results from regressing fund fees on market betas for subsamples of mutual funds. All the regressions are estimated for funds with betas larger than one. *Beta* is an estimate of the slope from the market model for fund returns. *(0,1) Alternative practices* indicator equals one if the fund engages in at least one of the activities such as borrowing money, short-selling, or trading options and futures. *(0,1) Fund beta>portfolio beta* indicator equals one if the difference between the fund beta and its stock holdings beta is larger than 0.05. *Stock portfolio PES* is a value-weighted average proportional effective spread of the fund holdings. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. The results are presented separately for funds with different values of *(0,1) Alternative practices* and *(0,1) Fund beta>portfolio beta* indicator variables, and for funds where *Stock portfolio PES* is above and below the its median value. p-values for tests of differences between the coefficients across the subsamples are reported. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	(0,1) Alternative practices			*(0,1) Fund beta>portfolio beta*			*Stock portfolio PES*		
	1	0	1 vs. 0 (p-value)	1	0	1 vs. 0 (p-value)	Above the median	Below the median	Above vs. Below (p-value)
Beta	0.42***	0.35***	0.51	0.28***	0.27***	0.93	0.32***	0.36***	0.62
	(0.07)	(0.11)		(0.10)	(0.07)		(0.05)	(0.08)	
Observations	3,949	9,693		5,820	17,758		19,823	17,342	
R-squared	0.82	0.77		0.74	0.73		0.75	0.69	
Control variables	Yes	Yes		Yes	Yes		Yes	Yes	
Fund family fixed effects	Yes	Yes		Yes	Yes		Yes	Yes	
Month fixed effects	Yes	Yes		Yes	Yes		Yes	Yes	

Internet Appendix

Model: Proofs and Additional Results

We provide proofs for our theoretical results that are not contained in the main text, as well as additional results for generalized cases.

Proof of Proposition 1 Condition 1 of the Proposition follows from the main text. For condition 2, compare two different funds k and j with $\beta_k > \beta_j$. Let ω_i^{j*} be the optimal allocation for fund j, such that the related utility for investor i is

$$\omega_i^{j*}(\beta_j(\mu_M - \xi) + \xi - \phi_j) + R_f - \frac{\gamma_i}{2}\omega_i^{j*2}\beta_j^2\sigma_M^2 \tag{A.1}$$

according to (1). We compare this to the utility that fund k provides, which is

$$\omega_i^{k}(\beta_k(\mu_M - \xi) + \xi - \phi_k) + R_f - \frac{\gamma_i}{2}\omega_i^{k2}\beta_k^2\sigma_M^2. \tag{A.2}$$

Now choose the weight of the risky investment for fund k as $\omega_i^k = \omega_i^{j*}\frac{\beta_j}{\beta_k}$. Then we have $\omega_i^k < \omega_i^{j*}$ and the related utility is obtained as

$$\omega_i^{j*}(\beta_j(\mu_M - \xi) + \frac{\beta_j}{\beta_k}(\xi - \phi_k)) + R_f - \frac{\gamma_i}{2}\omega_i^{j*2}\beta_j^2\sigma_M^2. \tag{A.3}$$

Comparing (A.1) and (A.3), we see that fund k dominates fund j unless the fees fulfill the condition $\xi - \phi_j \geq \frac{\beta_j}{\beta_k}(\xi - \phi_k)$. Therefore, funds with $\phi_j > \frac{\beta_j}{\beta_k}\phi_k + \xi(1 - \frac{\beta_j}{\beta_k})$ are dominated.

General version of Proposition 2 We characterize the investor fund preference dependent on their risk aversion in Proposition 2, focusing on the case that condition (4) is fulfilled. Here, we provide the general version of this result:

Proposition 2'. *[Risk Aversion and Fund Preference] Investor i with borrowing bound l prefers fund j over fund k, with $\beta_j > \beta_k$, if and only if $\gamma_i < \overline{\gamma_{jk}}$, with*

$$\overline{\gamma_{jk}} = \begin{cases} \frac{\beta_j \widetilde{\mu_M} + \xi - \phi_j}{\beta_j^2 \sigma_M^2 l}, & \text{for } \beta_j(\beta_k - \beta_j)^2 \widetilde{\mu_M} = \phi_j \left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2 \phi_k + \xi(\beta_j^2 - \beta_k^2) \\ 2\frac{\widetilde{\mu_M}(\beta_j - \beta_k) - (\phi_j - \phi_k)}{(\beta_j^2 - \beta_k^2)\sigma_M^2 l}, & \text{for } \beta_j(\beta_k - \beta_j)^2 \widetilde{\mu_M} < \phi_j \left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2 \phi_k + \xi(\beta_j^2 - \beta_k^2) \\ \frac{\beta_k \beta_j \widetilde{\mu_M} - \sqrt{(\beta_j(\phi_k - \xi) - \beta_k(\phi_j - \xi))(-2\beta_k \beta_j \widetilde{\mu_M} + \beta_j(\phi_k - \xi) + \beta_k(\phi_j - \xi))} - \beta_j(\phi_k - \xi)}{\beta_k^2 \beta_j \sigma_M^2 l}, & \\ & \text{for } \beta_j(\beta_k - \beta_j)^2 \widetilde{\mu_M} > \phi_j \left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2 \phi_k + \xi(\beta_j^2 - \beta_k^2). \end{cases} \tag{A.4}$$

Note that in the third case, the risk aversion "cutoff" value $\overline{\gamma_{jk}}$ depends non-linearly on the fund fees, and a numerical solution of the model is required in this case.

Proof of Proposition 2' To prove the Proposition, we simply compare the value of the objective in (1) for two funds j and k with $\beta_j > \beta_k$ for an investor with risk aversion γ_i and borrowing bound l. After inserting the optimal weights $\omega_i^{j^*} = \min\{\frac{\mu_j - \phi_j}{\gamma_i \sigma_j^2}, l\}$ and $\omega_i^{k^*} = \min\{\frac{\mu_k - \phi_k}{\gamma_i \sigma_k^2}, l\}$, the result for the different cases follows from standard calculations.

Proof of Proposition 3 We prove the Proposition by assuming the contrary. Suppose that $\overline{\gamma_{j_2 j_1}} \geq \overline{\gamma_{j_1 k}}$ holds for certain funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$. According to Proposition 2', that means that fund j_2 is preferred over j_1 by all investors with $\gamma_i < \overline{\gamma_{j_2 j_1}}$, and that investors with $\gamma_i \geq \overline{\gamma_{j_1 k}}$ prefer fund k over j_1 or are indifferent between them. As $\overline{\gamma_{j_2 j_1}} \geq \overline{\gamma_{j_1 k}}$, this implies that there is no level of risk aversion for which the corresponding investors prefer fund j_1 over all other funds, such that j_1 does not survive in equilibrium.

Similarly, suppose that $\overline{\gamma_{j_2 k}} \geq \overline{\gamma_{j_1 k}}$ holds for certain funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$. According to Proposition 2', that means that investors with $\overline{\gamma_{j_1 k}} \leq \gamma_i \leq \overline{\gamma_{j_2 k}}$ prefer fund j_2 over k and prefer k over j_1 or are indifferent between them. This implies that the "cutoff" $\overline{\gamma_{j_2 j_1}}$ below which investors prefer j_2 over j_1 lies in $\overline{\gamma_{j_1 k}} \leq \overline{\gamma_{j_2 j_1}} < \overline{\gamma_{j_2 k}}$. Furthermore, investors with $\gamma_i \geq \overline{\gamma_{j_1 k}}$ prefer fund k over j_1 or are indifferent between them. As $\overline{\gamma_{j_1 k}} \leq \overline{\gamma_{j_2 j_1}}$, there is no level of risk aversion for which the corresponding investors prefer fund j_1 over all other funds, such that j_1 does not survive in equilibrium.

As we assume that fund j_1 exists in equilibrium, it follows that $\overline{\gamma_{j_2 j_1}} < \overline{\gamma_{j_1 k}}$ and $\overline{\gamma_{j_2 k}} < \overline{\gamma_{j_1 k}}$ for all funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$.

Equilibrium solution for linear case If the second case of Proposition 2' applies (i.e., condition (4) is fulfilled) for all funds, then the first order conditions obtained from the fund manager optimization problems (2) constitute a linear equation system $A\phi = b$, with $\phi = (\phi_0, \phi_1, \ldots, \phi_J)'$ being the vector of fund fees. We explicitly state the matrix A and vector b, considering the case $\xi = 0$ and $\psi = 1$ for ease of exposition. In this case, A is the tridiagonal matrix

$$A = \begin{pmatrix} \frac{2}{\beta_0^2-\beta_1^2} & \frac{1}{\beta_1^2-\beta_0^2} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{1}{\beta_1^2-\beta_0^2} & \frac{2(\beta_0^2-\beta_2^2)}{(\beta_2^2-\beta_1^2)(\beta_1^2-\beta_0^2)} & \frac{1}{\beta_2^2-\beta_1^2} & \ddots & & & \vdots \\ 0 & \frac{1}{\beta_2^2-\beta_1^2} & \frac{2(\beta_1^2-\beta_3^2)}{(\beta_3^2-\beta_2^2)(\beta_2^2-\beta_1^2)} & \frac{1}{\beta_3^2-\beta_2^2} & \ddots & & \vdots \\ \vdots & \ddots & \frac{1}{\beta_3^2-\beta_2^2} & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \ddots & \ddots & \frac{1}{\beta_J^2-\beta_{J-1}^2} \\ 0 & \cdots & \cdots & \cdots & 0 & \frac{1}{\beta_J^2-\beta_{J-1}^2} & \frac{2}{\beta_{J-1}^2-\beta_J^2} \end{pmatrix}, \tag{A.5}$$

and

$$b = \begin{pmatrix} \overline{\Gamma}\sigma_M^2/2 - \widetilde{\mu_M}\frac{1}{\beta_0+\beta_1} \\ \mu_M \frac{\beta_2-\beta_0}{(\beta_0+\beta_1)(\beta_1+\beta_2)} \\ \mu_M \frac{\beta_3-\beta_1}{(\beta_1+\beta_2)(\beta_2+\beta_3)} \\ \vdots \\ \widetilde{\mu_M}\frac{1}{\beta_{J-1}+\beta_J} - \underline{\Gamma}\sigma_M^2/2 \end{pmatrix}. \tag{A.6}$$

Clearly, the solution of the linear equation system can be obtained analytically for an arbitrary number of funds, as specified by J.

Constant values in equilibrium solution (9) For the case $J = 3$, with fund 1 being the market ETF, the constants A_1, A_2, B_1, B_2, and C are given by

$$\begin{aligned}
A_1 &= (\beta_2 - \beta_M)(\beta_2 + 2\beta_3 - \beta_M), \\
A_2 &= (2\beta_3^2 - \beta_2^2 - \beta_2\beta_3 + (\beta_2 + \beta_3)\beta_M - 2\beta_M^2), \\
B_1 &= (\beta_2 + \beta_3)(\beta_2 - \beta_M)(\beta_2 + \beta_M), \\
B_2 &= (\beta_2 + \beta_3)(2\beta_3^2 - \beta_2^2 - \beta_M^2), \\
C &= \frac{\beta_3 - \beta_2}{4\beta_3^2 - \beta_2^2 - 3\beta_M^2}.
\end{aligned} \tag{A.7}$$

Note that all the constants are positive since all betas are greater than one and ordered by their magnitudes.

Proof of Proposition 4 To prove the Proposition, let us first state the equilibrium solution for general ψ and $\bar{l}$, which is given by:

$$\begin{aligned}
\phi_2 - \phi_M &= \frac{1}{C}(A_1\widetilde{\mu_M} - \frac{\bar{l}}{2(1 + (\bar{l} - 1)\psi)} B_1 \underline{\Gamma}\sigma_M^2), \\
\phi_3 - \phi_2 &= \frac{1}{C}(A_2\widetilde{\mu_M} - \frac{\bar{l}}{2(1 + (\bar{l} - 1)\psi)} B_2 \underline{\Gamma}\sigma_M^2).
\end{aligned} \tag{A.8}$$

Note that for $\psi = 1$ or $\bar{l} = 1$, we are back to the solution stated in (9). Part (i) of the Proposition then follows for the sufficient condition $\underline{\Gamma}\sigma_M^2 < \widetilde{\mu_M}/\beta_3$, as described in the main text. For (ii), observe that $\frac{\partial \frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}}{\partial \psi} < 0$ and $\frac{\partial \frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}}{\partial \bar{l}} > 0$, from which the result follows. Part (iii) is an immediate implication of part (i), as gross alphas $\alpha'_j = \mu_j - \beta_j \mu_M$ are either zero in the model (for the CAPM case) or themselves falling in betas (for the BAB case).

Additional Empirical Results

Table A1: Relation between Fund Market Beta, Fund Fees, and Intensity of Fund Offerings

This table presents the results from regressing mutual fund fees on fund market beta and measures of intensity of fund offerings, separately for funds with betas larger than one and smaller than one. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. *Number of funds per beta bin* is the number of funds (in thousands) with the value of beta falling into each 0.1 bin (e.g., fund with betas between 0.8–0.9 are in a bin, funds with betas between 1.1–1.2 are in another bin, etc.) in a specific month. *HHI per beta bin* is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Share class level		Fund level		Share class level		Fund level	
	Beta>1	*Beta*<1	*Beta*>1	*Beta*<1	*Beta*>1	*Beta*<1	*Beta*>1	*Beta*<1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Beta	0.33***	-0.09	0.28***	-0.00	0.52***	0.04	0.39***	0.03
	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.05)	(0.05)	(0.06)
N of funds per beta bin	-0.04**	0.03**	-0.07	-0.02				
	(0.02)	(0.02)	(0.05)	(0.05)				
HHI per beta bin					-0.29**	0.28*	-0.19*	0.29**
					(0.15)	(0.15)	(0.11)	(0.12)
Observations	511,810	476,797	219,912	218,872	511,810	476,797	219,912	218,872
R-squared	0.48	0.47	0.69	0.66	0.48	0.47	0.69	0.66
Control variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table A2: Relation between Fund Market Beta and Fund Fees across Distribution Channels

This table presents the results from regressing mutual fund fees on fund market beta separately for direct-sold and broker-sold fund share classes. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. A fund share class is considered *Direct-sold* if it charges no front or back load, and has an annual distribution fee ("12b-1" fee) of no more than 25 basis points; otherwise it is considered *Broker-sold*. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Share class level			
	Beta>1		*Beta*<1	
	Broker-sold	*Direct-sold*	*Broker-sold*	*Direct-sold*
	(1)	(2)	(3)	(4)
Beta	0.45***	0.39***	0.06	0.09
	(0.06)	(0.04)	(0.06)	(0.06)
Observations	202,644	309,159	185,480	291,305
R-squared	0.71	0.46	0.69	0.47
Control variables	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes

Table A3: Relation between Fund Market Beta and Fund Fees: Fund Style Fixed Effects Regressions

This table presents the results from regressing mutual fund fees on fund market beta and fund style fixed effects, separately for funds with betas larger than one and smaller than one. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Beta* is an estimate of the slope from the market model for fund returns. Fund style fixed effects are defined based on the fund Lipper classification. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Share class level		Fund level	
	Beta>1	*Beta*<1	*Beta*>1	*Beta*<1
	(1)	(2)	(3)	(4)
Beta	0.27***	0.02	0.21***	0.04
	(0.06)	(0.06)	(0.05)	(0.06)
Observations	494,236	447,855	205,488	193,260
R-squared	0.49	0.52	0.72	0.72
Control variables	Yes	Yes	Yes	Yes
Fund style fixed effects	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes