Online Appendix for "Paying for Beta: Leverage Demand and Asset Management Fees"

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March 2022

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Part A: Theoretical Model Details

A.1 Investor Choice and Fund Assets Under Management

For the model setting outlined in Section 2 of the paper, we examine the investors' investment choices, which ultimately determine the funds' assets under management. We condition on the asset managers' beta choices β_j; we are able to solve for their endogenous betas after calculating the funds' assets and equilibrium fund fees conditional on betas. We assume that an asset manager j survives in equilibrium only if some investors prefer j over all other managers in the universe.

Investor Choice Without loss of generality, suppose that investor i decides to invest with asset manager j. Then the first order condition for the weight of the risky investment is

$$\widetilde{\omega}_i^j = \frac{\mu_j - \phi_j}{\gamma_i \sigma_j^2}, \tag{A1}$$

and i chooses her investment to be $\omega_i^{j^*} = \min\{\widetilde{\omega}_i^j, l\}$ due to the borrowing constraint.

We describe the investor's choice between different asset managers and show first that investors do not invest with asset managers whose fees are too high, either in an absolute sense or relative to other managers. All proofs are provided in Appendix A.3.

Proposition A1. *[Dominated Funds] Investors do not invest into funds j with*

1. $\phi_j \geq \mu_j$ or with
2. $\phi_j > \frac{\beta_j}{\beta_k}\phi_k + \xi(1 - \frac{\beta_j}{\beta_k})$ for a fund k with $\beta_j < \beta_k$.

In Proposition A1, we provide necessary conditions for asset managers to have positive assets under management and to survive in equilibrium. The first part states that no investor is willing to invest with a manager whose expected after-fee excess return $\mu_j - \phi_j$ is smaller or equal zero. In the second part, we lay out the basic logic for our main result. In particular, the fees of asset managers with smaller betas are bounded by the fees of higher-beta managers. For illustration, consider the case in which the CAPM holds in the asset market ($\xi = 0$). In this case, equilibrium fees must be non-decreasing in betas since investors can always synthesize a lower-beta fund by investing in a fund with higher beta and holding a cash position. This argument does not apply the other way round: investors cannot synthesize a high-beta fund by a leveraged investment in a lower-beta fund due to borrowing constraints.

As a result, asset managers with low betas cannot charge higher fees than asset managers with higher betas.[1]

We next characterize the investment decision of an individual investor given her risk aversion γ_i. By comparing the levels of utility provided by two funds j and k, with $\beta_j > \beta_k$ and optimal investment weights ω_i^{j*} and ω_i^{k*}, we show that investors prefer fund j over k if their risk aversion is below a certain threshold, which we denote by $\overline{\gamma_{jk}}$. For notational ease, define $\widetilde{\mu_M} = \mu_M - \xi$.

Proposition A2. *[Risk Aversion and Fund Preference] Investor i with borrowing bound l prefers fund j over fund k, with $\beta_j > \beta_k$, if and only if $\gamma_i < \overline{\gamma_{jk}}$, with*

$$\overline{\gamma_{jk}} = \begin{cases} \frac{\beta_j \widetilde{\mu_M} + \xi - \phi_j}{\beta_j^2 \sigma_M^2 l}, & \text{for } \beta_j(\beta_k - \beta_j)^2 \widetilde{\mu_M} = \phi_j\left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2 \phi_k + \xi(\beta_j^2 - \beta_k^2) \\ 2\frac{\widetilde{\mu_M}(\beta_j - \beta_k) - (\phi_j - \phi_k)}{(\beta_j^2 - \beta_k^2)\sigma_M^2 l}, & \text{for } \beta_j(\beta_k - \beta_j)^2 \widetilde{\mu_M} < \phi_j\left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2 \phi_k + \xi(\beta_j^2 - \beta_k^2) \\ \frac{\beta_k \beta_j \widetilde{\mu_M} - \sqrt{(\beta_j(\phi_k - \xi) - \beta_k(\phi_j - \xi))(-2\beta_k\beta_j\widetilde{\mu_M} + \beta_j(\phi_k - \xi) + \beta_k(\phi_j - \xi))} - \beta_j(\phi_k - \xi)}{\beta_k^2 \beta_j \sigma_M^2 l}, & \\ & \text{for } \beta_j(\beta_k - \beta_j)^2 \widetilde{\mu_M} > \phi_j\left(\beta_k^2 + \beta_j^2\right) - 2\beta_j^2 \phi_k + \xi(\beta_j^2 - \beta_k^2). \end{cases} \tag{A2}$$

We illustrate this result in Figure A1 and show the combinations of investor risk aversion γ_i and fee ϕ_j for which an asset manager j dominates the market index fund with $\beta_M = 1$ and $\phi_M = 0$. In both plots, the yellow (light, not meshed) area stands for the region in which the asset manager with $\beta = 1.3$ is preferred to the market index fund. Investors with very low risk aversion are willing to pay a lot for leverage and prefer the high-beta asset manager over the market index fund even if the asset manager charges a very high fee. The fee at which the high-beta fund j is preferred declines in investor risk aversion.

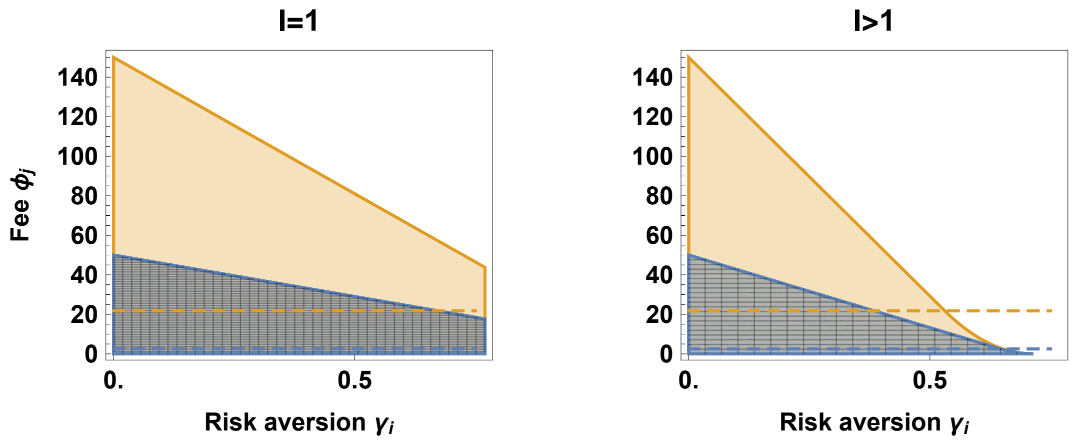
Comparing this to the blue (dark, meshed) area—the region in which an asset manager with lower beta ($\beta = 1.1$) is preferred to the market index fund—highlights the effect on fees across asset managers with different betas. The yellow area overlays the blue area: the manager with $\beta = 1.3$ can set higher fees and still be strictly preferred by some investors over the market index fund. As risk aversion declines, the investor is willing to pay significantly more to the high-beta asset manager even if the low-beta manager is available.

Finally, we graphically illustrate the role of the tightness of leverage constraints. The left plot describes the choice of an investor who faces strict constraints ($l = 1$), while the right plot presents the more relaxed case ($l > 1$). In the strict case, investors cannot obtain

[1] If ξ is substantially greater than zero, the restriction on fees through Condition 2 of Proposition A1 is somewhat relaxed, but we show that in the model equilibrium fees increase in beta particularly for $\beta > 1$.

Figure A1: Risk Aversion, Fund Beta, and Willingness to Pay

This figure presents constellations of investor i's risk aversion γ_i and fund j's fee ϕ_j for which j is preferred over the market index fund with $\beta_M = 1$ and $\phi_M = 0$. The blue (dark, meshed) region presents the relation for a fund with $\beta_j = 1.1$, the yellow (light, not meshed) region presents the relation for a fund with $\beta_j = 1.3$. The left plot describes investors who face strict borrowing constraints, the right plot presents the case of less constrained investors with $l = 1.75$. The dashed lines stand for the ϕ_j value above which the second case of Proposition A2 applies and the region is linear. Parameters are set according to the CAPM case in Table A1.



leverage by any means. As a result, even investors with moderate risk aversion prefer high-beta asset managers over the market index fund if the fee is not too extreme.

We extend this logic further and show that in equilibrium, investors sort across managers depending on their betas, and the corresponding investor clienteles are formed based on risk aversion.

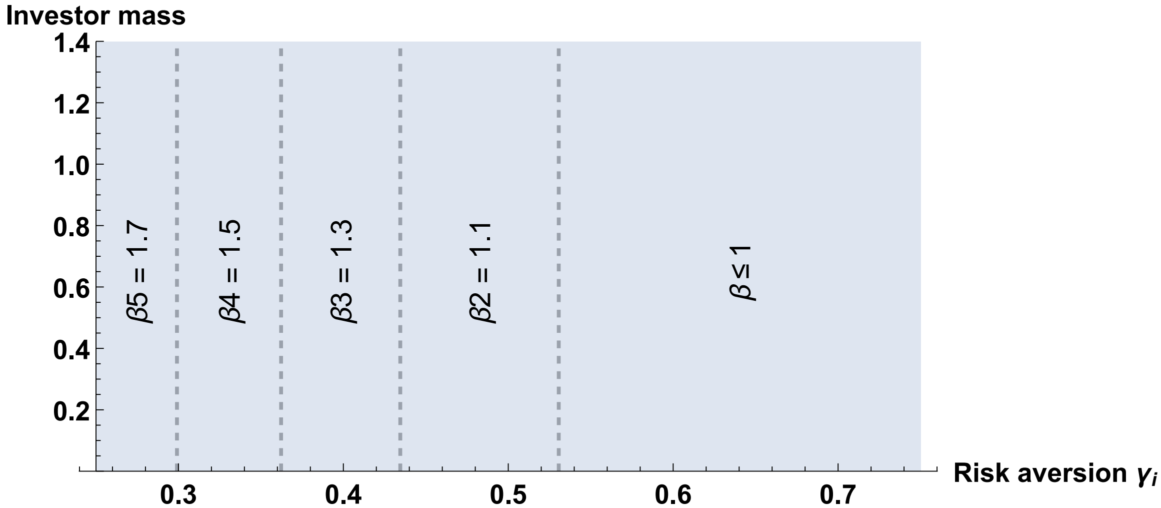
Proposition A3. *[Investor Clienteles] For all funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$, it must be that $\overline{\gamma_{j_2 j_1}} < \overline{\gamma_{j_1 k}}$ and $\overline{\gamma_{j_2 k}} < \overline{\gamma_{j_1 k}}$ in equilibrium. Asset managers with higher betas are chosen by investors with lower risk aversion.*

We illustrate this result in Figure A2. In the equilibrium, asset managers with different betas offer their services to different types of investors. In particular, investors with the lowest risk aversion choose the asset manager with the highest beta, up to a certain cutoff point, after which the second-least risk-averse clientele chooses the fund with the second-highest beta, and so on.

Using the results from Propositions A2 and A3, we can compute the assets under management (AUM) of fund j, dependent on the fee ϕ_j. In particular, the AUM are given

Figure A2: Distribution of Investors across Funds

This figure illustrates how investors sort across asset managers based on their risk aversion γ_i, given four managers with $\beta > 1$, the market index fund with $\beta_1 = \beta_M = 1$, and possible additional funds with $\beta < 1$. Fund fees are set exemplarily to $(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5) = (0, 11, 38, 67, 100)$ basis points. All the investors face relaxed borrowing constraints ($l = 1.75$). Further parameters are set according to the CAPM case in Table A1.



by

$$AUM_j(\phi_j) = \int_{\overline{\gamma}_{j+1,j}}^{\overline{\gamma}_{j,j-1}} \min\{\frac{\mu_j - \phi_j}{\gamma_i \sigma_j^2}, l\} f(\gamma_i) d\gamma_i, \tag{A3}$$

where the integration bounds are defined in line with Proposition A2, and $f(.)$ is the probability density for the risk aversion in the investor population. We utilize the fact that asset manager j attracts investors whose risk aversion is below the threshold $\overline{\gamma}_{j,j-1}$ at which the manager with the next-lower beta is dominated, but larger than the value $\overline{\gamma}_{j+1,j}$ at which manager j is dominated by the manager with the next-higher beta.

A.2 Equilibrium Fund Fees Conditional on Betas

We solve for the equilibrium fund fees conditional on the asset managers' beta choices. Given fund betas β_j, an equilibrium is a combination of fees ϕ_j for the asset managers such that, for optimal investor choices, fee revenues are maximized for all asset managers (according to the optimization problems (1) and (2) in the paper). To explicitly solve for the model equilibrium in fees, we need to make an assumption on the probability distribution of γ_i. We assume that γ_i is equally distributed on $[\underline{\Gamma}, \overline{\Gamma}]$. The equilibrium can then be efficiently computed numerically after inserting (A3) into the objective in (2). Moreover, we obtain an analytical solution if one of the first two cases of Proposition A2 applies and the investors' borrowing constraints are binding. In that case, the first order conditions obtained from the fund manager optimization problems (2) constitute a linear equation system $A\phi = b$, where ϕ is the vector of all fund fees, and A is a tridiagonal matrix.

Solution for Baseline Case with Four Funds We explicitly demonstrate and explore the equilibrium solution for a scenario of four funds with betas $0 < \beta_0 < \beta_1 = \beta_M = 1 < \beta_2 < \beta_3$ for the case that the second case of Proposition A2 applies and that the investors' borrowing constraints are binding. Moreover, we start with $\xi = 0$ and $\psi = 1$ for ease of exposition. Since there is perfect supply-side competition for market index funds with $\beta_1 = \beta_M = 1$, the index fund fee ϕ_M equals the marginal management cost which is zero. Proposition A1 then implies that for $\xi = 0$ the fee ϕ_0 for the asset manager with $\beta_0 < 1$ is zero too; otherwise, it would always be optimal for investors to invest in the market index fund and cash in order to replicate the fund with β_0 at zero fees. The same argument holds for potential additional asset managers with beta smaller than one, such that fees become flat in betas for $\beta < 1$.

We next solve for the fees ϕ_3 and ϕ_2 of the funds with $\beta_3 > \beta_2 > 1$, which are set by their managers under monopolistic competition. The revenue maximization problems (2), in which we insert the assets under management computed according to (A3) with uniformly distributed γ_i and binding borrowing constraints, are obtained as

$$\begin{aligned}
&\max_{\phi_2}\ \phi_2 \cdot \frac{1}{\overline{\Gamma} - \underline{\Gamma}} \left(2\frac{\widetilde{\mu_M}(\beta_2 - \beta_M) - (\phi_2 - \phi_M)}{(\beta_2^2 - \beta_M^2)\sigma_M^2} - 2\frac{\widetilde{\mu_M}(\beta_3 - \beta_2) - (\phi_3 - \phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} \right), \\
&\max_{\phi_3}\ \phi_3 \cdot \frac{1}{\overline{\Gamma} - \underline{\Gamma}} \left(2\frac{\widetilde{\mu_M}(\beta_3 - \beta_2) - (\phi_3 - \phi_2)}{(\beta_3^2 - \beta_2^2)\sigma_M^2} - \underline{\Gamma} \right).
\end{aligned} \tag{A4}$$

Given the fees, all investors with low enough risk aversion, down to the lowest risk aversion $\underline{\Gamma}$, prefer the β_3 manager over the β_2 manager. These investors invest with the β_3 manager since there are no managers with higher beta. Another group of investors invests with the β_2 manager. These investors are more risk-averse relative to the first group and prefer the β_2 manager over the β_3 manager. At the same time, these investors still have low enough risk aversion such that they do not invest in the market index fund. The rest of the investors chooses the market index fund. Given the investor demand, the fund managers maximize revenues by setting the appropriate fees.

Taking the derivatives of the fund managers' objective functions by ϕ_2 and ϕ_3, respectively, and setting them to zero, yields the corresponding first order conditions

$$\begin{gathered} 2\frac{\widetilde{\mu_M}(\beta_2-\beta_M)-(2\phi_2-\phi_M)}{(\beta_2^2-\beta_M^2)\sigma_M^2}-2\frac{\widetilde{\mu_M}(\beta_3-\beta_2)-(\phi_3-2\phi_2)}{(\beta_3^2-\beta_2^2)\sigma_M^2}=0, \\ 2\frac{\widetilde{\mu_M}(\beta_3-\beta_2)-(\phi_3-2\phi_2)}{(\beta_3^2-\beta_2^2)\sigma_M^2}-\underline{\Gamma}=0. \end{gathered} \tag{A5}$$

Since $\phi_M = 0$, we can solve the given system of two equations for the two fee variables, ϕ_2 and ϕ_3. The solution can be written as

$$\begin{aligned} \phi_2-\phi_M &= \frac{1}{C}(A_1\widetilde{\mu_M}-\frac{1}{2}B_1\underline{\Gamma}\sigma_M^2), \\ \phi_3-\phi_2 &= \frac{1}{C}(A_2\widetilde{\mu_M}-\frac{1}{2}B_2\underline{\Gamma}\sigma_M^2), \end{aligned} \tag{A6}$$

where the constants A_1, A_2, B_1, B_2, C are given by

$$\begin{aligned} A_1 &= (\beta_2-\beta_M)(\beta_2+2\beta_3-\beta_M), \\ A_2 &= (2\beta_3^2-\beta_2^2-\beta_2\beta_3+(\beta_2+\beta_3)\beta_M-2\beta_M^2), \\ B_1 &= (\beta_2+\beta_3)(\beta_2-\beta_M)(\beta_2+\beta_M), \\ B_2 &= (\beta_2+\beta_3)(2\beta_3^2-\beta_2^2-\beta_M^2), \\ C &= \frac{\beta_3-\beta_2}{4\beta_3^2-\beta_2^2-3\beta_M^2}. \end{aligned} \tag{A7}$$

Note that all the constants are positive since all betas are greater than one and ordered by their magnitudes.

Heterogeneous Borrowing Constraints and "Betting Against Beta" When investors have heterogeneous borrowing constraints (i.e., $0 < \psi < 1$) and the security market line in the asset market is relatively flat ("betting-against-beta" case, $\xi > 0$), the analytical solution for ϕ_2 and ϕ_3 has the same form as in (A6) and can be obtained following analogous calculations. In particular, the solution for general $0 < \psi < 1$ is obtained by replacing $\frac{1}{2}$ with $\frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}$:

$$\begin{aligned} \phi_2-\phi_M &= \frac{1}{C}(A_1\widetilde{\mu_M}-\frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}B_1\underline{\Gamma}\sigma_M^2), \\ \phi_3-\phi_2 &= \frac{1}{C}(A_2\widetilde{\mu_M}-\frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}B_2\underline{\Gamma}\sigma_M^2). \end{aligned} \tag{A8}$$

Note that for $\psi = 1$ or $\bar{l} = 1$, we are back to the solution stated in (A6). The $\xi > 0$ case enters the solution (A6) and (A8) through $\widetilde{\mu_M}$ and further requires that we explicitly solve for ϕ_0 along the same lines, as the fee of the β_0 fund is not bounded to zero by ϕ_M in that case.

General Solution for Linear Case The analytical solution for the case that the second case of Proposition A2 applies for all funds and all investors' borrowing constraints are binding extends to scenarios of J funds. More precisely, the first order conditions obtained from the fund manager optimization problems (2) constitute a linear equation system $A\phi = b$, with $\phi = (\phi_0, \phi_1, \ldots, \phi_J)'$ being the vector of fund fees. We explicitly state the matrix A and vector b, considering the case $\xi = 0$ and $\psi = 1$ for ease of exposition. In this case, A is the tridiagonal matrix

$$A = \begin{pmatrix} \frac{2}{\beta_0^2-\beta_1^2} & \frac{1}{\beta_1^2-\beta_0^2} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{1}{\beta_1^2-\beta_0^2} & \frac{2(\beta_0^2-\beta_2^2)}{(\beta_2^2-\beta_1^2)(\beta_1^2-\beta_0^2)} & \frac{1}{\beta_2^2-\beta_1^2} & \ddots & & & \vdots \\ 0 & \frac{1}{\beta_2^2-\beta_1^2} & \frac{2(\beta_1^2-\beta_3^2)}{(\beta_3^2-\beta_2^2)(\beta_2^2-\beta_1^2)} & \frac{1}{\beta_3^2-\beta_2^2} & \ddots & & \vdots \\ \vdots & \ddots & \frac{1}{\beta_3^2-\beta_2^2} & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & \ddots & \ddots & \frac{1}{\beta_J^2-\beta_{J-1}^2} \\ 0 & \cdots & \cdots & \cdots & 0 & \frac{1}{\beta_J^2-\beta_{J-1}^2} & \frac{2}{\beta_{J-1}^2-\beta_J^2} \end{pmatrix}, \tag{A9}$$

and

$$b = \begin{pmatrix} \overline{\Gamma}\sigma_M^2/2 - \widetilde{\mu_M}\frac{1}{\beta_0+\beta_1} \\ \mu_M \frac{\beta_2-\beta_0}{(\beta_0+\beta_1)(\beta_1+\beta_2)} \\ \mu_M \frac{\beta_3-\beta_1}{(\beta_1+\beta_2)(\beta_2+\beta_3)} \\ \vdots \\ \widetilde{\mu_M}\frac{1}{\beta_{J-1}+\beta_J} - \underline{\Gamma}\sigma_M^2/2 \end{pmatrix}. \tag{A10}$$

Clearly, the solution of the linear equation system can be obtained analytically for an arbitrary number of funds, as specified by J.

A.3 Proofs

We provide the proofs for our theoretical results in the following.

Proof of Proposition A1 Condition 1 of the Proposition follows directly from the investor's optimal investment weight according to equation (A1). For condition 2, compare two different funds k and j with $\beta_k > \beta_j$. Let $\omega_i^{j^*}$ be the optimal allocation for fund j, such that the related utility for investor i is

$$\omega_i^{j^*}(\beta_j(\mu_M - \xi) + \xi - \phi_j) + R_f - \frac{\gamma_i}{2}\omega_i^{j^*2}\beta_j^2\sigma_M^2 \tag{A11}$$

according to (1) in the paper. We compare this to the utility that fund k provides, which is

$$\omega_i^{k}(\beta_k(\mu_M - \xi) + \xi - \phi_k) + R_f - \frac{\gamma_i}{2}\omega_i^{k^2}\beta_k^2\sigma_M^2. \tag{A12}$$

Now choose the weight of the risky investment for fund k as $\omega_i^k = \omega_i^{j^*}\frac{\beta_j}{\beta_k}$. Then we have $\omega_i^k < \omega_i^{j^*}$ and the related utility is obtained as

$$\omega_i^{j^*}(\beta_j(\mu_M - \xi) + \frac{\beta_j}{\beta_k}(\xi - \phi_k)) + R_f - \frac{\gamma_i}{2}\omega_i^{j^*2}\beta_j^2\sigma_M^2. \tag{A13}$$

Comparing (A11) and (A13), we see that fund k dominates fund j unless the fees fulfill the condition $\xi - \phi_j \geq \frac{\beta_j}{\beta_k}(\xi - \phi_k)$. Therefore, funds with $\phi_j > \frac{\beta_j}{\beta_k}\phi_k + \xi(1 - \frac{\beta_j}{\beta_k})$ are dominated.

Proof of Proposition A2 To prove the Proposition, we simply compare the value of the objective in (1) in the paper for two funds j and k with $\beta_j > \beta_k$ for an investor with risk aversion γ_i and borrowing bound l. After inserting the optimal weights $\omega_i^{j^*} = \min\{\frac{\mu_j - \phi_j}{\gamma_i\sigma_j^2}, l\}$ and $\omega_i^{k^*} = \min\{\frac{\mu_k - \phi_k}{\gamma_i\sigma_k^2}, l\}$, the result for the different cases follows from standard calculations.

Proof of Proposition A3 We prove the Proposition by assuming the contrary. Suppose that $\overline{\gamma_{j_2 j_1}} \geq \overline{\gamma_{j_1 k}}$ holds for certain funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$. According to Proposition A2, that means that fund j_2 is preferred over j_1 by all investors with $\gamma_i < \overline{\gamma_{j_2 j_1}}$, and that investors with $\gamma_i \geq \overline{\gamma_{j_1 k}}$ prefer fund k over j_1 or are indifferent between them. As $\overline{\gamma_{j_2 j_1}} \geq \overline{\gamma_{j_1 k}}$, this implies that there is no level of risk aversion for which the corresponding investors prefer fund j_1 over all other funds, such that j_1 does not survive in equilibrium.

Similarly, suppose that $\overline{\gamma_{j_2 k}} \geq \overline{\gamma_{j_1 k}}$ holds for certain funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$. According to Proposition A2, that means that investors with $\overline{\gamma_{j_1 k}} \leq \gamma_i \leq \overline{\gamma_{j_2 k}}$ prefer fund j_2 over k and prefer k over j_1 or are indifferent between them. This implies that the "cutoff" $\overline{\gamma_{j_2 j_1}}$ below which investors prefer j_2 over j_1 lies in $\overline{\gamma_{j_1 k}} \leq \overline{\gamma_{j_2 j_1}} < \overline{\gamma_{j_2 k}}$. Furthermore, investors with $\gamma_i \geq \overline{\gamma_{j_1 k}}$ prefer fund k over j_1 or are indifferent between them. As $\overline{\gamma_{j_1 k}} \leq \overline{\gamma_{j_2 j_1}}$, there is no level of risk aversion for which the corresponding investors prefer fund j_1 over all other funds, such that j_1 does not survive in equilibrium.

As we assume that fund j_1 exists in equilibrium, it follows that $\overline{\gamma_{j_2 j_1}} < \overline{\gamma_{j_1 k}}$ and $\overline{\gamma_{j_2 k}} < \overline{\gamma_{j_1 k}}$ for all funds j_1, j_2, k with $\beta_{j_2} > \beta_{j_1} > \beta_k$.

Proof of Proposition 1 The proof of our main proposition is in great parts given by the derivations provided in Appendix A.2. For the equilibrium fees solution (A8), note that the $\widetilde{\mu_M}$ term is greater than the negative σ_M^2 term in both expressions for all relevant combinations of the given parameters.[2] Therefore, Part (i) of the Proposition directly follows. For (ii), observe that $\frac{\partial \frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}}{\partial \psi} < 0$ and $\frac{\partial \frac{\bar{l}}{2(1+(\bar{l}-1)\psi)}}{\partial \bar{l}} > 0$, from which the result follows. Part (iii) is an immediate implication of part (i), as gross alphas $\alpha_j' = \mu_j - \beta_j \mu_M$ are either zero in the model (for the CAPM case) or themselves falling in betas (for the BAB case).

A.4 Model Calibration

We calibrate the model as specified in Table A1. In particular, we consider a scenario in which the CAPM holds in the asset market as well as a "betting-against-beta" scenario with a flatter security market line. We also consider different scenarios of more or less leverage-constrained investors.

[2] A sufficient condition is $\underline{\Gamma}\sigma_M^2 < \widetilde{\mu_M}/\beta_3$. In our model calibration (see Table A1), it is $\underline{\Gamma}\sigma_M^2 = 0.01$. The condition is therefore fulfilled as long as $\beta_3 < 5$ in the CAPM case and as long as s $\beta_3 < 3.5$ in the BAB case, respectively.

Table A1: Model Parameters, Borrowing-Constraint Scenarios, and Fund Betas

This table presents the parameters we use to calibrate the model for different scenarios. Parameters describing the expected return and volatility of the stock market are set to the standard values of 5% and 20% per year, respectively. We consider a CAPM case and a betting-against-beta (BAB) case with a flatter security market line. The investors' absolute risk aversion in our calibrated model is uniformly distributed between 0.25 and 0.75. For the investors' borrowing constraints, we consider a "more constrained" scenario in which 80% of the investors are strictly borrowing-constrained and 20% can obtain a leverage of 1.5, and a "less constrained" scenario in which 20% of investors are strictly constrained and 80% can obtain a leverage of 1.75.

Parameter		Value	
	Stock market		
Expected stock market return	μ_M	0.05	
Stock market volatility	σ_M	0.2	
		CAPM	BAB
Betting-against-beta parameter	ξ	0.0	0.015
	Fund investors		
Highest absolute risk aversion	$\overline{\Gamma}$	0.75	
Lowest absolute risk aversion	$\underline{\Gamma}$	0.25	
		More constrained	Less constrained
Mass of strictly borrowing-constr. investors ($l = 1$)	ψ	0.8	0.2
Max. leverage for less borrowing-constrained investors	$\bar{l}$	1.5	1.75

Part B: Supplementary Datasets and Additional Empirical Results

B.1 Supplementary Datasets

We compile four supplementary datasets. The first dataset includes proxies for fund management costs based on the funds' portfolio characteristics. These data allow us to analyze whether fees of high-beta funds are partly driven by higher stock trading or equity valuation costs (see Section B.2.5). We collect data on funds' portfolio holdings from Thomson Reuters and calculate stock characteristics using information from the CRSP and Compustat databases. As a proxy for stock trading costs, we compute proportional effective spreads (PES) (Lesmond, Schill, and Zhou, 2004; Hasbrouck, 2009; Novy-Marx and Velikov, 2016) for the funds' portfolios. As proxies for equity valuation costs, we compile portfolio characteristics from Sheng, Simutin, and Zhang (2020) such as asset tangibility, asset growth, operating profitability, analyst coverage, and idiosyncratic volatility. Details on this dataset are provided in Appendix B.1.1.

The second supplementary dataset is based on funds' N-SAR filings and provides detailed information on fund investment practices. These data allow us to examine how funds obtain leverage in Section 5 of the paper. To compile this dataset, we collect information from N-SAR forms, which funds are required to file semiannually with the SEC. We collect these filings from the EDGAR database using an automated scraping algorithm and match them by the fund name to our main fund-level sample. Of particular interest for our analysis is the information provided in Item 70 of the N-SAR forms and its sub-items. These items entail detailed information on whether a fund has engaged in various investment practices during the reporting period, such as borrowing money, short-selling, and futures and options trading. Details on the dataset on investment practices are provided in Appendix B.1.2.

The third dataset includes a sample of leveraged ETFs, which we use to examine the effects of leveraged ETF entry on the relation between beta and fees. We obtain the dataset from a comprehensive sample of leveraged ETFs compiled by Lu and Qin (2021).

The fourth dataset contains style-based index funds which invest in value, growth, small-cap, and large-cap stocks. This dataset is used to analyze the effect of HML and SMB betas on fund fees. To collect these data, we search for phrases such as "value", "growth", "large", "small", "small-cap", "large-cap", "micro-cap" in the names of index funds and classify these funds into factor-based investment styles based on their names. The descriptive statistics of this dataset in Appendix Table B4 show that HML and SMB betas of style-based index funds are consistent with the name-based definitions.

B.1.1 Supplementary Dataset: Portfolio Characteristics

We use a dataset of fund portfolio holding characteristics to generate proxies for fund management costs in our robustness tests. Using quarterly holdings data from Thomson Reuters, we collect data on portfolio holdings at the end of each quarter and calculate stock characteristics using information from the CRSP and Compustat databases. To match the quarterly reporting of fund holdings, we use portfolio characteristics from the last month of each quarter (i.e., in March, June, September, and December) unless stated otherwise. We aggregate characteristics at the fund-level by value-weighting individual stock characteristics in the fund's portfolio.

We use the proportional effective spread (PES) as a proxy for a stock's trading costs (Lesmond, Schill, and Zhou, 2004; Hasbrouck, 2009; Novy-Marx and Velikov, 2016). In each reporting month (again the last month of each quarter) and for each stock in the fund's portfolio, we calculate the daily stock PES using the closing prices as

$$PES_{it} = \frac{2|Price_{it} - 0.5 \times (Bid_{it} + Ask_{it})|}{Price_{it}}. \tag{B1}$$

We use portfolio characteristics from Sheng, Simutin, and Zhang (2020) as proxies for equity valuation costs. Asset tangibility is defined as the ratio of the amount of plant, property, and equipment to the firm's total assets. Asset growth is the natural logarithm of the ratio of total assets at the end of a quarter to total assets at the end of the previous quarter. Operating profitability is the company's revenue reduced by the costs of goods sold and by various expenses (selling, general, administrative, and interest expenses), relative to the firm's book equity (Fama and French, 2015). Analyst coverage is the number of analysts who provide earnings forecasts for a stock. Idiosyncratic volatility is the standard deviation of the residual from the market model. The summary statistics for fund portfolio characteristics are reported in Table B3.

B.1.2 Supplementary Dataset: Fund Investment Practices

When we examine how funds provide leverage in Section 5 of the paper, we employ an additional dataset on fund investment practices. To compile this dataset, we collect information from N-SAR filings, which are required for registered investment management companies.

The filings are made available by the SEC in a standardized electronic format through the EDGAR database. Item 70 of the N-SAR form provides detailed information on whether the fund has engaged in various investment practices during the reporting period. We collect these filings using an automated scraping algorithm and match them by the fund name to our main fund-level sample. The matching of fund names is done algorithmically and is validated by manual checks.[3]

Overall, 70% of the funds in our main CRSP sample have at least one N-SAR filing matched. N-SAR forms are filed semiannually, and we match the last month of the reporting period to our main sample.[4] We ultimately have 26,831 fund-month observations matched over the period 1995–2016. For the matched funds, we observe one N-SAR filing record per year on average, suggesting a fund-month matching rate of approximately 50%.

We focus on a number of selected investment practices which are associated with leverage. The fund's answers to the questions (Qs) in the N-SAR filings precisely reveal whether the fund engages in these alternative practices. In particular, we follow Warburton and Simkovic (2019) and collect information on whether the fund: (1) borrows money (Q 70.O); (2) engages in short-selling (Q 70.R); (3) trades in options on individual equities or stock indices (Qs 70.B and 70.D); or (4) trades in stock index futures or options on stock index futures (Qs 70.F and 70.H). We create an indicator variable that equals one if the fund engages in any of these alternative investment practices, as well as similarly defined indicator variables for each practice separately.

We also construct an indicator variable which equals one if the difference between the fund's beta and the beta of the stock portfolio is larger than 0.05. We require the difference to be slightly larger than zero due to potential errors in the estimation of betas.[5] A positive difference indicates the presence of instruments other than stocks to lever up fund portfolio, providing another proxy for usage of alternative investment practices. The

[3]We match fund names based on the Levenshtein distance, a leading string matching metric in computer science. While the algorithm assigns a match to every fund name in principle, we treat entries with a matching score below 95 (out of 100) as unmatched. This strategy ensures that there are no false positive matches. An example for a match with a score of 95 is 'Phoenix Strategic Equity Series Fund: Phoenix-Seneca Equity Opportunities Fund' in CRSP vs. 'PHOENIX STRATEGIC EQUITY SERIES FUND: PHOENIX EQUITY OPPORTUNITIES FUND' in the N-SAR filings.

[4]The N-SAR filings do not specify during which particular months of a reporting period a certain investment practice was used, but funds do not change their investment practices very frequently. For the investment practices of interest, as defined below, the 1-period (half-year) autocorrelation is 0.82, and fund fixed effects explain 71% of the variation. Our results are almost identical when we match the information from an N-SAR filing to *all* months of the reporting period.

[5]The stock betas are estimated using the same procedure as for the fund betas, described in Section 3.2 of the paper.

combined summary statistics for the dataset on investment practices are discussed in great detail in Section 5.1 of the paper.

B.2 Robustness of Main Results

B.2.1 Robustness to Fund Offerings across Betas

We discuss a number of robustness checks for our main results. We first examine the robustness of our findings to the variation in the number of fund offerings with beta, as documented in Figure B1. Our concern is that fees may increase with beta due to the decline in the number of alternative choices, and not due to the effects of leverage demand. To address this concern, we construct two measures to capture the intensity of fund offerings within different ranges of betas. The first measure counts the overall number of funds for each 0.1-wide beta bin in a specific month (e.g., funds with betas between 0.8-0.9 are assigned to one bin, and funds with betas between 1.1-1.2 are assigned to another bin, etc.). The second measure computes the Herfindahl-Hirschman Index (HHI) for each beta bin in a specific month, where a fund's market share is defined as the fund's assets under management (AUM) divided by the AUM of all the funds in the same beta bin. We use the value of the respective intensity measure for all funds in the corresponding beta bin.

We estimate equation (5) in the paper including the intensity measures in our specifications and report the results in Panel A of Table B5. For brevity, we only present the estimated coefficients on beta and on the interaction between beta and the indicator for beta being larger than one, which directly correspond to the specifications from Table 2. The detailed results are reported in Appendix Tables B6. Our main results remain unchanged, and the estimates of the coefficients on beta are quantitatively and qualitatively similar to the estimates from Table 2.

B.2.2 Robustness to Differences in Investors across Distribution Channels

We next explore whether the effects of beta on fees vary across distribution channels. Since the funds sold to investors via brokers have higher fees and higher beta relative to direct-sold funds (Del Guercio and Reuter, 2014), our results could be confounded by the differences in clienteles across these channels. To mitigate this concern, we examine the relation between beta and fees separately for direct-sold and for broker-sold funds. We follow Sun (2021) and consider a fund share class to be direct-sold if it charges no front or back load, and has an annual distribution fee ("12b-1 fee") of no more than 25 basis points; otherwise, a fund share class is considered as broker-sold.

We report the estimated coefficients on beta in Panel B of Table B5. The effect of beta on fees is quantitatively similar and statistically significant across the channels, suggesting that our results are robust to the differences in clienteles between direct-sold and broker-sold funds.

B.2.3 Robustness to Demand for Style Investing

We next examine the effects of fund styles on our main results. Since investors seek for exposure to different types of stocks, fund fees may vary across styles (Gil-Bazo and Ruiz-Verdú, 2009). If funds in investment categories (styles) with high-beta stocks have higher fees, the relation between beta and fees may reflect the demand for style investing rather than the demand for leverage. To account for this, we add fund style fixed effects (interacting with month fixed effects) to our main specifications. We define fund styles using the Lipper classification of the U.S. equity funds, which constitutes the basis for the CRSP fund style classifications. The estimated coefficients on beta in Panel C of Table B5 show that the effect of beta on fees holds within styles.

B.2.4 Robustness to Definition of Fees

We next study the robustness of our results to the definition of fund fees. In our main tests, we combine fund expense ratios as well as front and back loads to approximate the total costs of holding a fund. However, the information on loads is more poorly populated in the CRSP data, especially in the early years of the sample. To address this issue, we estimate equation (5) from the paper when fees include only expense ratios. The results, presented in Panel D of Table B5, are almost identical to the baseline estimates, suggesting that the asymmetric relation between beta and fees is not driven by loads or by incomplete data.

B.2.5 Robustness to Trading and Valuation Costs

Finally, we check whether our results are driven by fund trading or equity valuation costs. If high-beta stocks are more expensive to trade or to evaluate, then high fees of high-beta funds can be driven by higher management costs and not by beta itself. To analyze whether our results are confounded by variation in management costs, we make use of our portfolio characteristics dataset introduced in Section B.1 (see also Appendix B.1.1). In particular, we use the proportional effective spread (PES) as a proxy for a stock's trading costs (Lesmond, Schill, and Zhou, 2004; Hasbrouck, 2009; Novy-Marx and Velikov, 2016). We also use multiple portfolio characteristics from Sheng, Simutin, and Zhang (2020) as proxies for costs of equity valuation, adding these variables to our baseline specification in the sample of funds with beta larger than one.

We report the results in Appendix Table B10. Briefly, the relation between beta and fees is highly robust to the inclusion of a variety of proxies for trading and valuation costs. In line with Sheng, Simutin, and Zhang (2020), funds that invest in hard-to-evaluate firms charge higher fees.[6] However, fund beta itself has a distinct effect on fees which is robust to the inclusion of all the cost proxies together. Therefore, we conclude that our results are unlikely to be driven by trading or valuation costs.

[6]As highlighted by Sheng, Simutin, and Zhang (2020), these firms may be younger, have less tangible assets, lower analyst coverage, and faster asset growth.

B.3 Additional Empirical Results

Figure B1: Distribution of Fund Market Beta

This figure presents the empirical distribution of funds across market betas. The bars show the fraction of funds for each level of beta. *Beta* is an estimate of the slope from the market model for fund returns.

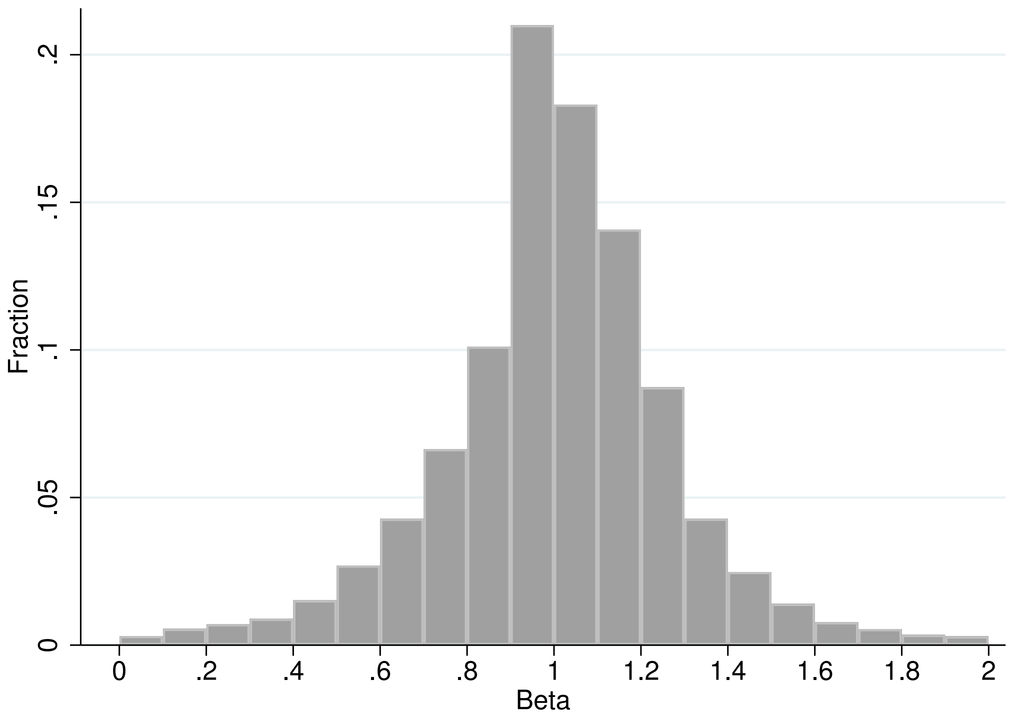


Table B1: Summary Statistics: Fund Share Classes

This table presents summary statistics for the sample of fund-month observations over the period 1991–2016 at the fund share class level. The fund characteristics are from the CRSP mutual fund database. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. β^{MKT} is an estimate of the slope from the market model for fund returns. α^{GROSS} and α^{NET} are annualized estimates of the intercept from the market models for fund gross returns and fund net returns, respectively. *TNA* is the fund total net assets. *Age* is the fund age in months. $1_{Passive}$ indicator equals one if a fund is passively managed. 1_{ETF} indicator equals one if a fund is an ETF. 1_{Retail} indicator equals one if a share class is offered to retail investors. The last column reports the R^2 of the regressions of variables on fund share class fixed effects.

	N	Mean	SD	Within SD	5%	25%	50%	75%	95%	R^2 - fund share class FE
Fee (%)	1,006,767	1.60	0.74	0.04	0.42	1.00	1.57	2.15	2.76	0.96
β^{MKT}	1,006,767	1.01	0.25	0.05	0.58	0.90	1.01	1.15	1.40	0.70
α^{GROSS} from CAPM (%)	1,006,767	1.55	5.76	1.75	-5.22	-0.97	1.04	3.54	10.39	0.38
α^{NET} from CAPM (%)	1,006,767	-0.05	5.76	1.75	-7.02	-2.52	-0.44	1.92	8.67	0.38
TNA ($MM)	1,006,767	785	3,921		1	7	83	385	2,915	
Age (months)	1,006,767	149.57	113.91		59	84	119	173	341	
$1_{Passive}$	1,006,767	0.07	0.26		0	0	0	0	1	
1_{ETF}	1,006,767	0.03	0.16		0	0	0	0	0	
1_{Retail}	1,006,767	0.68	0.47		0	0	1	1	1	

Table B2: Summary Statistics: Funds at Launch

This table presents summary statistics for the sample of fund-month observations over the period 1991–2016 at the fund level at the time of fund launch. The fund characteristics are from the CRSP mutual fund database. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. β^{MKT} is an estimate of the slope from the market model for fund returns. α^{GROSS} and α^{NET} are annualized estimates of the intercept from the market models for fund gross returns and fund net returns, respectively. *TNA* is the fund total net assets. $1_{Passive}$ indicator equals one if a fund is passively managed. 1_{ETF} indicator equals one if a fund is an ETF.

	N	Mean	SD	5%	25%	50%	75%	95%
Fee (%)	3,965	1.38	0.84	0.00	0.80	1.29	1.96	2.74
β^{MKT}	3,965	0.97	0.31	0.43	0.81	0.97	1.15	1.50
α^{GROSS} from CAPM (%)	3,965	1.98	7.05	-6.79	-1.03	1.54	4.79	13.53
α^{NET} from CAPM (%)	3,965	0.61	6.96	-8.13	-2.31	0.20	3.32	11.92
TNA	3,965	158.6	786.6	1	6.56	25.4	93.5	632
$1_{Passive}$	3,965	0.14	0.35	0	0	0	0	1
1_{ETF}	3,965	0.11	0.31	0	0	0	0	1

Table B3: Summary Statistics: Fund Portfolio Characteristics

This table presents summary statistics for fund portfolio characteristics. We aggregate characteristics at the fund level by value-weighting individual stock characteristics in the fund's portfolio. Panel A considers the full sample, Panel B the sample of funds with betas larger than one. β^{MKT} is an estimate of the slope from the market model for fund returns. *PES* is the proportional effective spread as a proxy for stock trading costs. *Asset tangibility* is the ratio of the amount of plant, property, and equipment to the firm's total assets. *Years since listed* is the number of years since the IPO. *Asset growth* is the natural logarithm of the ratio of total assets at the end of a quarter to total assets at the end of the previous quarter. *Operating profitability* is the company's revenue reduced by the costs of goods sold and by various expenses (selling, general, administrative, and interest expenses), relative to the firm's book equity. *Analyst coverage* is the number of analysts who provide earnings forecasts for a stock. *Idiosyncratic volatility* is the standard deviation of the residual from the market model.

Panel A: All funds	N	Mean	SD	5%	25%	50%	75%	95%
PES	58,136	0.195	0.219	0.039	0.064	0.132	0.244	0.564
Asset tangibility	59,896	0.235	0.102	0.118	0.182	0.221	0.266	0.412
Years since listed	59,711	13.505	4.668	5.682	10.255	13.526	16.659	21.109
Asset growth	59,934	0.027	0.024	-0.005	0.013	0.025	0.038	0.070
Operating profitability	59,501	0.085	0.035	0.029	0.067	0.087	0.105	0.132
Analyst coverage	60,080	12.304	3.965	8.257	9.963	10.839	13.940	20.167
Idiosyncratic volatility	58,144	0.089	0.030	0.053	0.067	0.083	0.104	0.146

Panel B: Funds with $\beta^{MKT} > 1$	N	Mean	SD	5%	25%	50%	75%	95%
PES	31,545	0.179	0.199	0.039	0.061	0.119	0.225	0.511
Asset tangibility	32,204	0.213	0.077	0.120	0.173	0.204	0.241	0.321
Years since listed	32,193	13.293	4.294	5.940	10.466	13.355	16.118	20.115
Asset growth	32,215	0.030	0.026	-0.005	0.014	0.026	0.042	0.076
Operating profitability	32,058	0.080	0.032	0.030	0.063	0.080	0.098	0.126
Analyst coverage	32,297	12.387	3.784	8.753	10.076	10.851	14.328	19.871
Idiosyncratic volatility	31,552	0.095	0.031	0.057	0.073	0.089	0.110	0.156

Table B4: Summary Statistics: Index Fund Betas

This table presents summary statistics for the corresponding factor betas of factor-based index funds. We collect this data by searching for phrases such as "SP500", "value", "growth", "large", "small", "small-cap", "large-cap", "micro-cap" in the names of index funds and classify these funds based on their names. β^{CAPM}_{SP500} is the CAPM beta of S&P 500 index funds. β^{HML}_{growth} and β^{HML}_{value} are the HML betas of growth and value index funds. β^{SMB}_{large} and β^{SMB}_{small} are the SMB betas of large-cap and small-cap index funds. The HML and SMB betas are estimated from the 3-factor model.

	N	Mean	SD	5%	25%	50%	75%	95%
β^{CAPM}_{SP500}	6,804	0.96	0.07	0.92	0.94	0.96	0.98	0.98
β^{HML}_{growth}	2,531	-0.28	0.09	-0.45	-0.33	-0.27	-0.23	-0.13
β^{HML}_{value}	3,504	0.41	0.25	0.13	0.26	0.32	0.48	0.91
β^{SMB}_{large}	7,523	-0.17	0.05	-0.24	-0.18	-0.15	-0.15	-0.13
β^{SMB}_{small}	4,916	0.81	0.14	0.56	0.74	0.83	0.87	0.98

Table B5: Relation between Fund Market Beta and Fund Fees: Baseline Robustness Checks

This table reports the results of robustness checks for the main tests presented in Table 2. Panel A reports the results from regressing fund fees on fund market beta and measures of the intensity of alternative fund offerings. Panel B reports the results from regressing fund fees on fund market beta separately for *Broker-sold* and *Direct-sold* fund share classes. Panel C reports the results from regressing fund fees on fund market beta by controlling for fund style fixed effects. Panel D reports the results from regressing fund expense ratios on fund market beta. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *Number of funds per beta bin* is the number of funds with the value of beta falling into each 0.1 bin (e.g., funds with betas between 0.8–0.9 are in a bin, funds with betas between 1.1–1.2 are in another bin, etc.) in a specific month. *HHI per beta* bin is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. A fund share class is considered *Direct-sold* if it charges no front or back load, and has an annual distribution fee ("12b-1" fee) of no more than 25 basis points; otherwise it is considered *Broker-sold*. Style fixed effects are defined based on the fund Lipper classification. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. Appendix Tables B6–B9 present the detailed results for these tests. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	(1)	(2)
Sample	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
Coefficient on	β^{MKT}	β^{MKT}
Panel A: Controlling for fund offerings (Tables B6)		
Add *N of funds per beta bin*	0.33***	-0.03
	(0.07)	(0.08)
Add *HHI per beta bin*	0.40***	0.02
	(0.05)	(0.06)
Panel B: Distribution channels (subsamples, share-class level) (Table B7)		
Within *Broker-sold*	0.34***	-0.07
	(0.06)	(0.06)
Within *Direct-sold*	0.32***	-0.13***
	(0.03)	(0.04)
Panel C: Fund styles (Table B8)		
Add Style fixed effects	0.21***	0.09
	(0.05)	(0.07)
Panel D: Expense ratios only (Table B9)		
	0.30***	-0.06
	(0.04)	(0.05)

Table B6: Relation between Fund Market Beta, Fund Fees, and Intensity of Fund Offerings

This table presents the results from regressing mutual fund fees on fund market beta and measures of intensity of fund offerings, separately for funds with betas larger than one and smaller than one. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. β^{MKT} is an estimate of the slope from the market model for fund returns. *Number of funds per beta bin* is the number of funds (in thousands) with the value of beta falling into each 0.1 bin (e.g., fund with betas between 0.8–0.9 are in a bin, funds with betas between 1.1–1.2 are in another bin, etc.) in a specific month. *HHI per beta bin* is the TNA-weighted Herfindahl-Hirschman Index (HHI) that is estimated for each 0.1 bin of beta in each month. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*				
	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
	(1)	(2)	(3)	(4)
β^{MKT}	0.33***	0.03	0.40***	-0.02
	(0.07)	(0.08)	(0.05)	(0.06)
N of funds per beta bin	-0.00	-0.00***		
	(0.00)	(0.00)		
HHI per beta bin			-0.12	0.19*
			(0.10)	(0.10)
Observations	188,563	178,579	188,563	178,579
R-squared	0.66	0.61	0.66	0.61
Control variables	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes

Table B7: Relation between Fund Market Beta and Fund Fees across Distribution Channels

This table presents the results from regressing mutual fund fees on fund market beta separately for direct-sold and broker-sold fund share classes. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. β^{MKT} is an estimate of the slope from the market model for fund returns. A fund share class is considered *Direct-sold* if it charges no front or back load, and has an annual distribution fee ("12b-1" fee) of no more than 25 basis points; otherwise it is considered *Broker-sold*. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*				
	$\beta^{MKT} > 1$		$\beta^{MKT} < 1$	
	Broker-sold	*Direct-sold*	*Broker-sold*	*Direct-sold*
	(1)	(2)	(3)	(4)
β^{MKT}	0.34***	0.32***	-0.07	-0.13***
	(0.06)	(0.03)	(0.06)	(0.04)
Observations	309,467	180,095	268,606	152,237
R-squared	0.49	0.67	0.49	0.70
Control variables	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes

Table B8: Relation between Fund Market Beta and Fund Fees: Fund Style Fixed Effects Regressions

This table presents the results from regressing mutual fund fees on fund market beta by further controlling for fund style fixed effects. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. β^{MKT} is an estimate of the slope from the market model for fund returns. Fund style fixed effects are defined based on the fund Lipper classification. All the specifications include fund family fixed effects, fund-style-month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*		
	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
	(1)	(2)
β^{MKT}	0.21***	0.09
	(0.05)	(0.067)
Observations	188,148	178,042
R-squared	0.73	0.71
Control variables	Yes	Yes
Fund family fixed effects	Yes	Yes
Fund style fixed effects × Month fixed effects	Yes	Yes

Table B9: Relation between Fund Market Beta and Fund Fees: Expense Ratios Only

This table presents the results from regressing mutual fund expense ratios on fund market beta separately for funds with betas larger than one and smaller than one. *Expense Ratio* is a fund's annual expense ratio. β^{MKT} is an estimate of the slope from the market model for fund returns. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Expense Ratio*		
	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
	(1)	(2)
β^{MKT}	0.30***	-0.06
	(0.04)	(0.05)
Observations	188,621	178,342
R-squared	0.75	0.70
Control variables	Yes	Yes
Fund family fixed effects	Yes	Yes
Month fixed effects	Yes	Yes

Table B10: Relation between Fund Market Beta and Fund Fees: Robustness to Trading and Valuation Costs

This table presents the results from regressing fund fees on fund market beta and proxies for trading and valuation costs. The proxies for a fund's trading and valuation costs are obtained by value-weighting individual stock characteristics in the fund's portfolio. All the regressions are estimated for funds with betas larger than one. β^{MKT} is an estimate of the slope from the market model for fund returns. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. *PES* is the proportional effective spread as a proxy for stock trading costs. *Asset tangibility* is the ratio of the amount of plant, property, and equipment to the firm's total assets. *Years since listed* is the number of years since the IPO. *Asset growth* is the natural logarithm of the ratio of total assets at the end of a quarter to total assets at the end of the previous quarter. *Operating profitability* is the company's revenue reduced by the costs of goods sold and by various expenses (selling, general, administrative, and interest expenses), relative to the firm's book equity. *Analyst coverage* is the number of analysts who provide earnings forecasts for a stock. *Idiosyncratic volatility* is the standard deviation of the residual from the market model. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
β^{MKT}	0.28***	0.29***	0.26***	0.29***	0.29***	0.30***	0.21***	0.23***
	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
PES	0.13**							0.08
	(0.06)							(0.06)
Asset tangibility		-0.06						-0.04
		(0.09)						(0.09)
Years since listed			-0.01***					-0.01**
			(0.00)					(0.00)
Asset growth				0.50**				0.16
				(0.23)				(0.20)
Operating profitability					-0.24			0.05
					(0.15)			(0.14)
Analyst coverage						-0.00		-0.00
						(0.00)		(0.00)
Idiosyncratic volatility							1.30***	0.73*
							(0.39)	(0.39)
Observations	31,522	32,182	32,172	32,194	32,037	32,275	31,527	31,258
R-squared	0.73	0.75	0.75	0.75	0.75	0.75	0.74	0.74
Control variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fund family fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table B11: Relation between Fund Market Beta and Tightness of Borrowing Constraints

This table presents information on the average change in fund market beta over constrained and unconstrained periods. We separately report the average changes for funds with beta larger than one and smaller than one. We also present the differences in the averages between constrained and unconstrained periods and the related p-values. The measures of borrowing constraint tightness include the BAB measure from Frazzini and Pedersen (2014), the ICR measure from He, Kelly, and Manela (2017), and the LCT measure from Boguth and Simutin (2018). The sample consists of months when the measure of tightness is either in the first quartile or in the fourth quartile of its distribution across time. β^{MKT} is an estimate of the slope from the market model for fund returns. $1_{Constrained}$ is defined for each measure separately and equals one if the BAB or ICR measures are in the first quartile of their distributions across time, and if the LCT measure is in the fourth quartile of its distribution across time. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively.

	$\Delta\beta^{MKT}$					
Measure of borrowing constraint tightness	BAB		ICR		LCT	
	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
$1_{Constrained} = 1$	-0.001	0.001	0.000	0.000	0.001	0.001
$1_{Constrained} = 0$	0.0011	0.000	0.002	0.000	-0.001	0.000
Difference	-0.002	0.001	-0.002	-0.001	0.002	0.001
P-value	0.368	0.526	0.393	0.644	0.311	0.622

Table B12: Relation between Fund Entries, Exits, and Tightness of Borrowing Constraints

This table presents information on fund entries and exits over constrained and unconstrained periods. We report the average fraction of fund entries and fund exits per period relative to the overall number of funds in that period, separately for funds with beta larger than one and smaller than one. We also present the differences in the averages between constrained and unconstrained periods and the related p-values. The measures of borrowing constraint tightness include the BAB measure from Frazzini and Pedersen (2014), the ICR measure from He, Kelly, and Manela (2017), and the LCT measure from Boguth and Simutin (2018). The sample consists of months when the measure of tightness is either in the first quartile or in the fourth quartile of its distribution across time. β^{MKT} is an estimate of the slope from the market model for fund returns. $1_{Constrained}$ is defined for each measure separately and equals one if the BAB or ICR measures are in the first quartile of their distributions across time, and if the LCT measure is in the fourth quartile of its distribution across time. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively.

Panel A:	Fraction of Fund Entries					
Measure of borrowing constraint tightness	BAB		ICR		LCT	
	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
$1_{Constrained} = 1$	0.0050	0.0049	0.0073	0.0074	0.0151	0.0146
$1_{Constrained} = 0$	0.0163	0.0134	0.0146	0.0143	0.0090	0.0072
Difference	-0.0149**	-0.0045	-0.0086	-0.0021	0.0019	0.0069
p-value	0.0348	0.4127	0.1539	0.6911	0.8407	0.1550

Panel B:	Fraction of Fund Exits					
Measure of borrowing constraint tightness	BAB		ICR		LCT	
	Beta>1	Beta<1	Beta>1	Beta<1	Beta>1	Beta<1
$1_{Constrained} = 1$	0.0034	0.0041	0.0053	0.0053	0.0042	0.0047
$1_{Constrained} = 0$	0.0026	0.0027	0.0023	0.0022	0.0027	0.0035
Difference	0.0005	0.0015	0.0023**	0.0045***	0.0004	0.0019
p-value	0.4971	0.1392	0.0135	0.0002	0.6115	0.1550

Table B13: Relation between Fund Market Beta and Fund Fees over Time

This table presents the results from regressing mutual fund fees on fund market beta for different sample periods, separately for funds with betas larger than one and smaller than one. *Fee* is the sum of the fund annual expense ratio and one-seventh of the sum of the front load and the back load. β^{MKT} is an estimate of the slope from the market model for fund returns. Only the coefficients on β^{MKT} are reported. All the specifications include fund family fixed effects, month fixed effects, and the full set of control variables. *,**, and *** denote statistical significance at 10%, 5%, and 1% levels, respectively. Standard errors double-clustered by fund family and month are in parentheses.

y = *Fee*	Fund level	
	$\beta^{MKT} > 1$	$\beta^{MKT} < 1$
Sample period	(1)	(2)
	Coefficient on β^{MKT}	
1995–2000	0.41***	-0.12
	(0.12)	(0.14)
2001–2005	0.40***	0.09
	(0.07)	(0.06)
2006–2010	0.29***	-0.16
	(0.06)	(0.10)
2011–2016	0.29***	-0.26***
	(0.06)	(0.08)

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